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NARODOWA STRATEGIA SPÓJNOŚCI

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„SIGNAL PROCESSING”

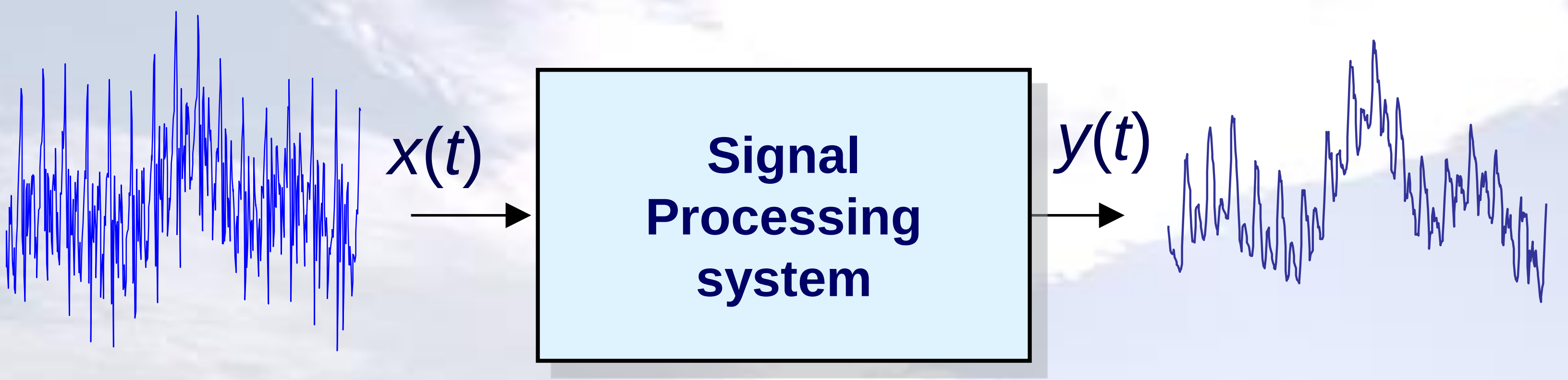
Prezentacja multimedialna współfinansowana przez Unię Europejską w ramach Europejskiego Funduszu Społecznego w projekcie pt.

„Innowacyjna dydaktyka bez ograniczeń - zintegrowany rozwój Politechniki Łódzkiej - zarządzanie Uczelnią, nowoczesna oferta edukacyjna i wzmacniania zdolności do zatrudniania osób niepełnosprawnych”

Signal processing systems

$$x(t) \rightarrow y(t)$$

?



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Signal processing systems

Continuous signal

$$y(t)=H(x(t))$$

$$x(t) \longrightarrow \boxed{\text{Continuous time system}} \longrightarrow y(t)$$

eg. megaphone - analog amplifier

t – continuous time variable

Signal processing systems

Discrete time signal

$y(n)=H(x(n))$

$x(n) \longrightarrow$ Discrete time system $\longrightarrow y(n)$

eg. reverberation (echo) model:

*y(n)=0.1*y(n-1) +x(n),*

→ *Digital filters*

n – discrete time variable *n* = 0, 1, … *N*,…

Signal processing systems - properties

1. With and without memory
2. Invertible and non-invertible
3. Causal and non-causal
4. Stable and unstable*
5. Linear and nonlinear
6. Time-invariant and time-variant

Examples?

Systems with and without memory

Output signal of a system without memory at instance *n* depends only on the input signal at the same time instance, eg:

$$y(n) = 3x(n) + 2x^2(n)$$

Output signal of a system with memory at instance *n* depends on the input signal at instances $k \neq n$, eg:

$$y(n) = \sum_{k=-\infty}^{n-1} x(k) + x(n) \longrightarrow y(n) = x(n-1) + x(n)$$

Causal systems

A system is causal if its output signal at instance *n* is dependent only on the input signal at instance *n* and/or previous instances, eg:

$$y(n) = x(n) + x(n-10)$$

$$\cancel{y(n) = x(n) - x(n+1)}$$

noncausal

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Stable/unstable system - quiz question

The system is considered to be **stable** if for input signal samples *x*(*n*) such that |*x*(*n*)|<*A* (where: *A* – is a finite constant) output signal samples *y*(*n*) satisfy |*y*(*n*)|<*B* (where: B – is also a finite constant) – this is so called bounded-input bounded-output (**BIBO**) stability.

1. Which of the system is stable?

a) $y(n) = x^3(n)$

b) $y(n) = \sum_{n=0}^{\infty} a^n\, u(n)$ where $u(n)$ is a sequence of unit pulses

c) $y(n) = k^2 x(-n)$ where k is a constant

d) $y(n) = \sum_{n=0}^{N} x(n)$

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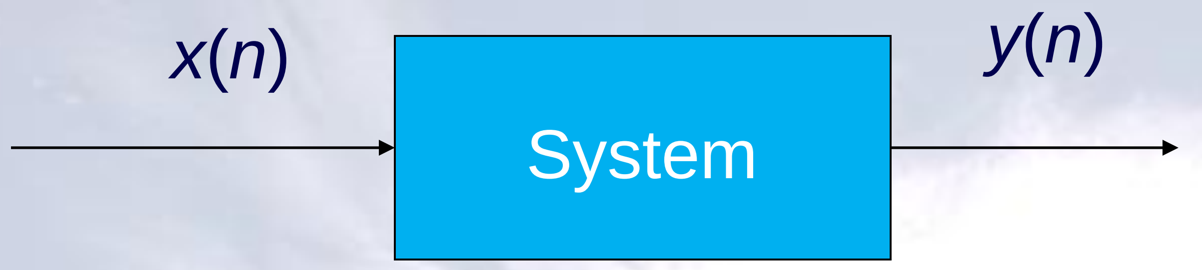
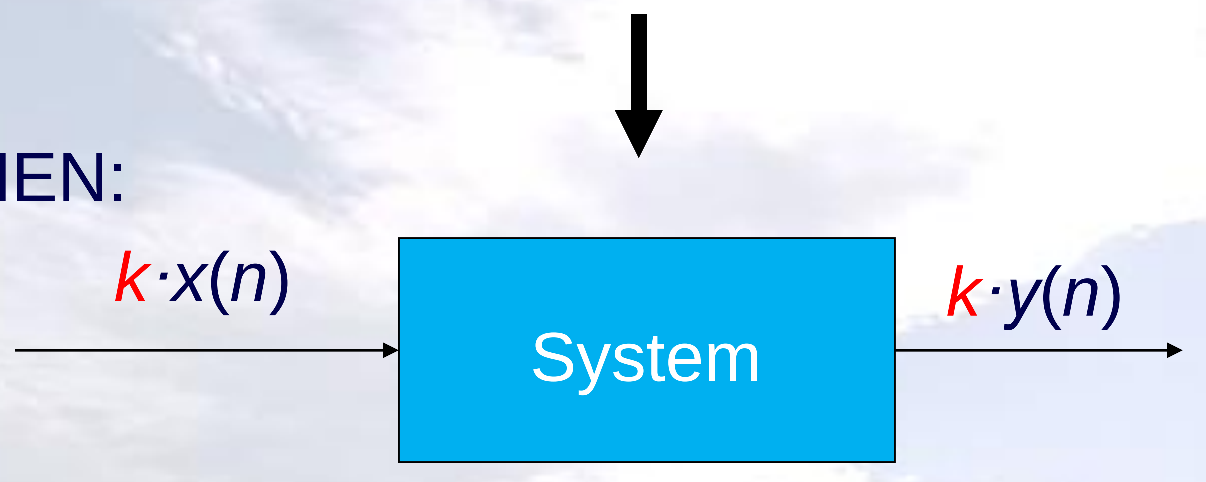
Homogeneous systems

IF:



THEN:



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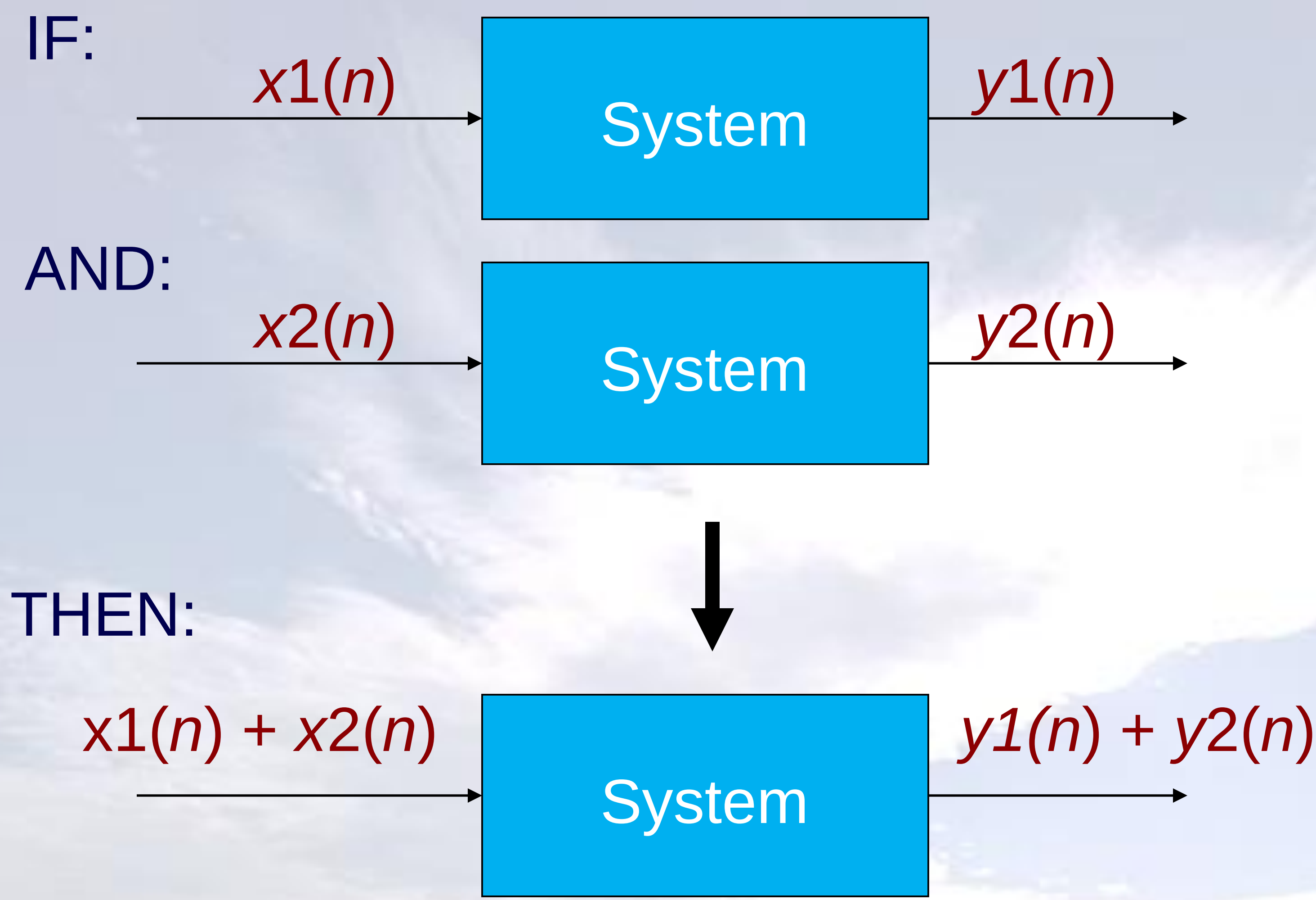
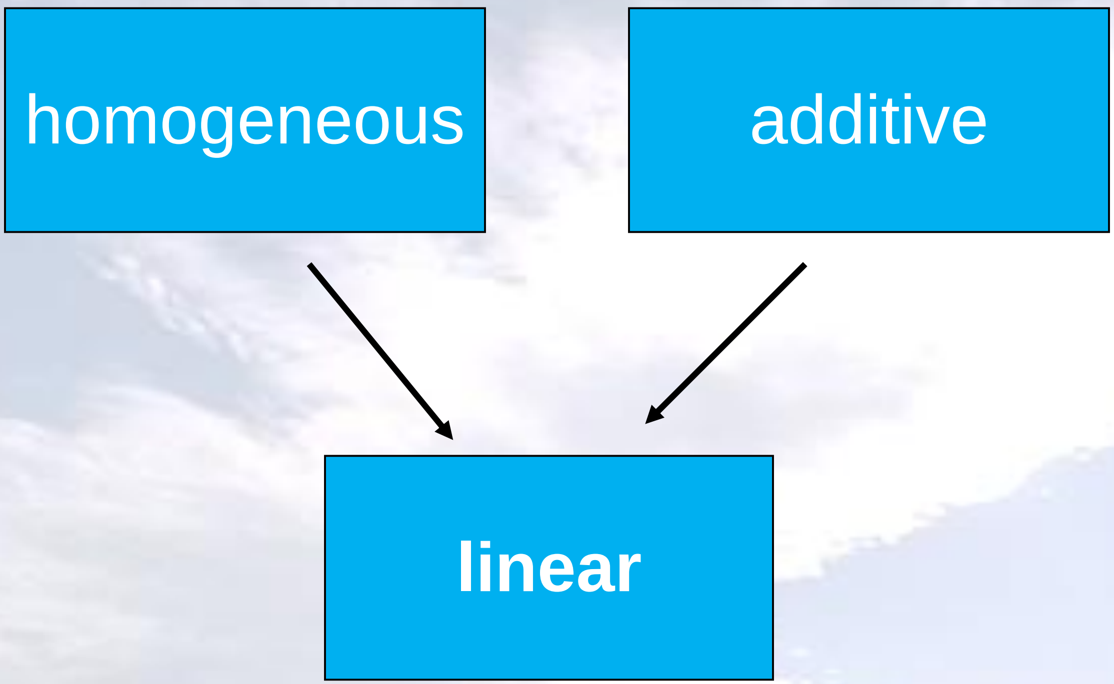
Additive systems



Linear systems

If the system is homogeneous and additive, it is linear.



Superposition principle

Linear systems fulfill the superposition principle:

$$\boxed{H\left[ax_1(n)+bx_2(n)\right]=aH\left[x_1(n)\right]+bH\left[x_2(n)\right]=ay_1(n)+by_2(n)}$$

ie. the response of a linear system to a sum of input signals is equal to the sum of the responses of the input signals.

Linear system example: $y(n)=3x(n)$

Nonlinear system example: $y(n)=x^2(n)$

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Linear systems

Is the following system: $y(n) = 2x(n) + 1$ linear?

Let: $x_1(n) = 2 \quad and \quad x_2(n) = 3$

NO!

$$y_1(n) = 2x_1(n) + 1 = 5 \qquad y_2(n) = 2x_2(n) + 1 = 7$$

However:

$$y_3(n) = 2[x_1(n) + x_2(n)] + 1 = 11 \neq y_1(n) + y_2(n) = 12$$

Conclusion: *The response of a linear system to zero input is…?*

Linear systems

Is the following system: $y(n) = 2x(n) + 1$ linear?

Remember:

$$\boxed{H\left[ax_1(n) + bx_2(n)\right] = ay_1(n) + by_2(n)}$$

$$H\left[ax_1(n) + bx_2(n)\right] = 2\left[ax_1(n) + bx_2(n)\right] + 1 = 2ax_1(n) + 2bx_2(n) + 1$$

But:

$$ay_1(n) + by_2(n) = 2ax_1(n) + 2bx_2(n) + a + b$$

Quiz questions

1. Which of these systems are linear? Show calculations.

a) $y(n) = 3x(n)$

b) $y(n) = \sum_{n=0}^{N} x(n)$

c) $y(n) = k^2 x(-n)$

d) $y(n) = x^2(n)$

e) $y(t) = a\log(x(t))$

f) $y(t) = \int_{o}^{t} x(\tau)d\tau$

Time invariant systems

Time invariant systems have the following property:

IF $y(n)$ is the response of the system to $x(n)$

THEN $y(n-k)$ is the response to $x(n-k)$.

$$\boxed{y(n) = H[x(n)] \Rightarrow y(n-k) = H[x(n-k)]}$$

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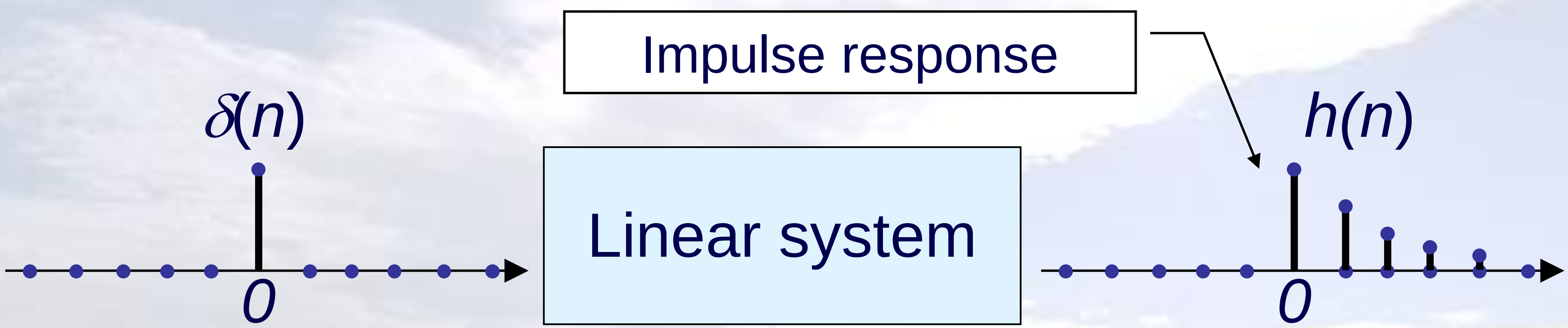
LTI systems

We will concentrate on the LTI systems.

We will show that for LTI systems,
the knowledge of the system response to the impulse $\delta(n)$ is enough to determine the response of the system to any discrete time signal.



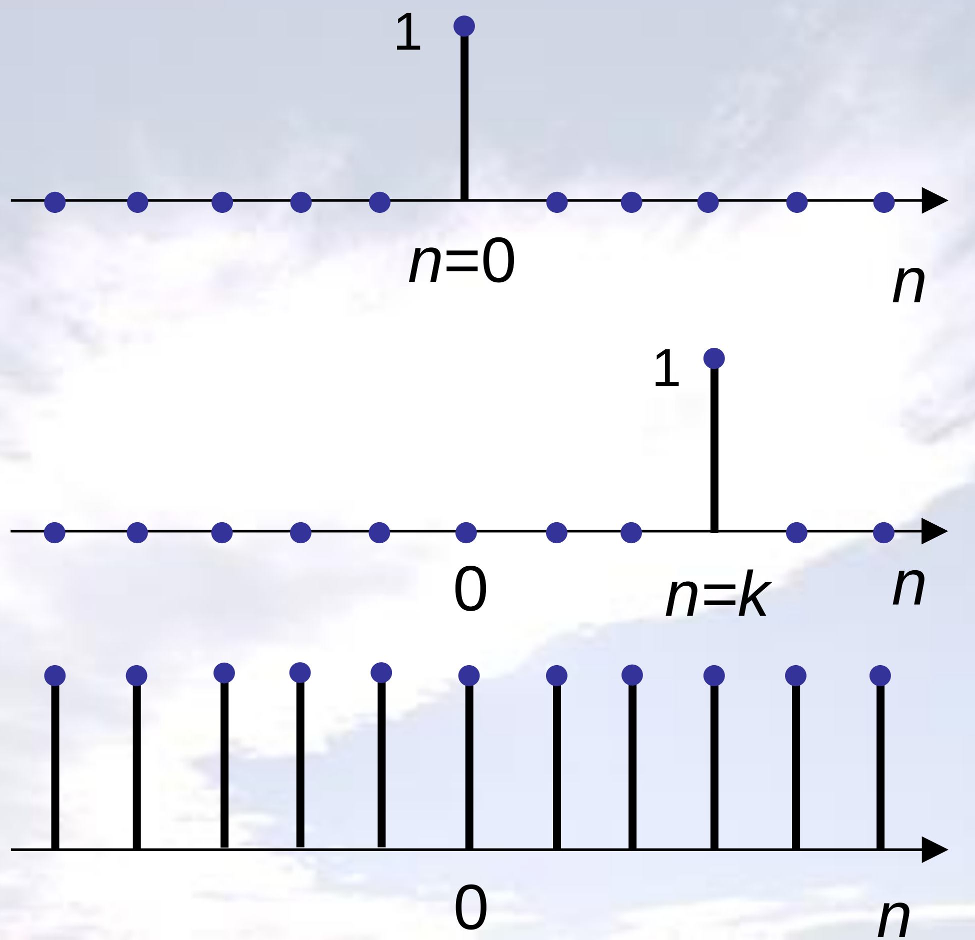
Continous vs discrete time signal (Dirac impulse series)

Unit impulse:

$$\delta(n)=\begin{cases}1 & for\;\; n=0\\ 0 & for\;\; n\neq 0\end{cases}$$

$$\delta(n-k)$$

$$\sum_{k=-\infty}^{k=\infty}\delta(n-k)$$



Continous vs discrete time signal

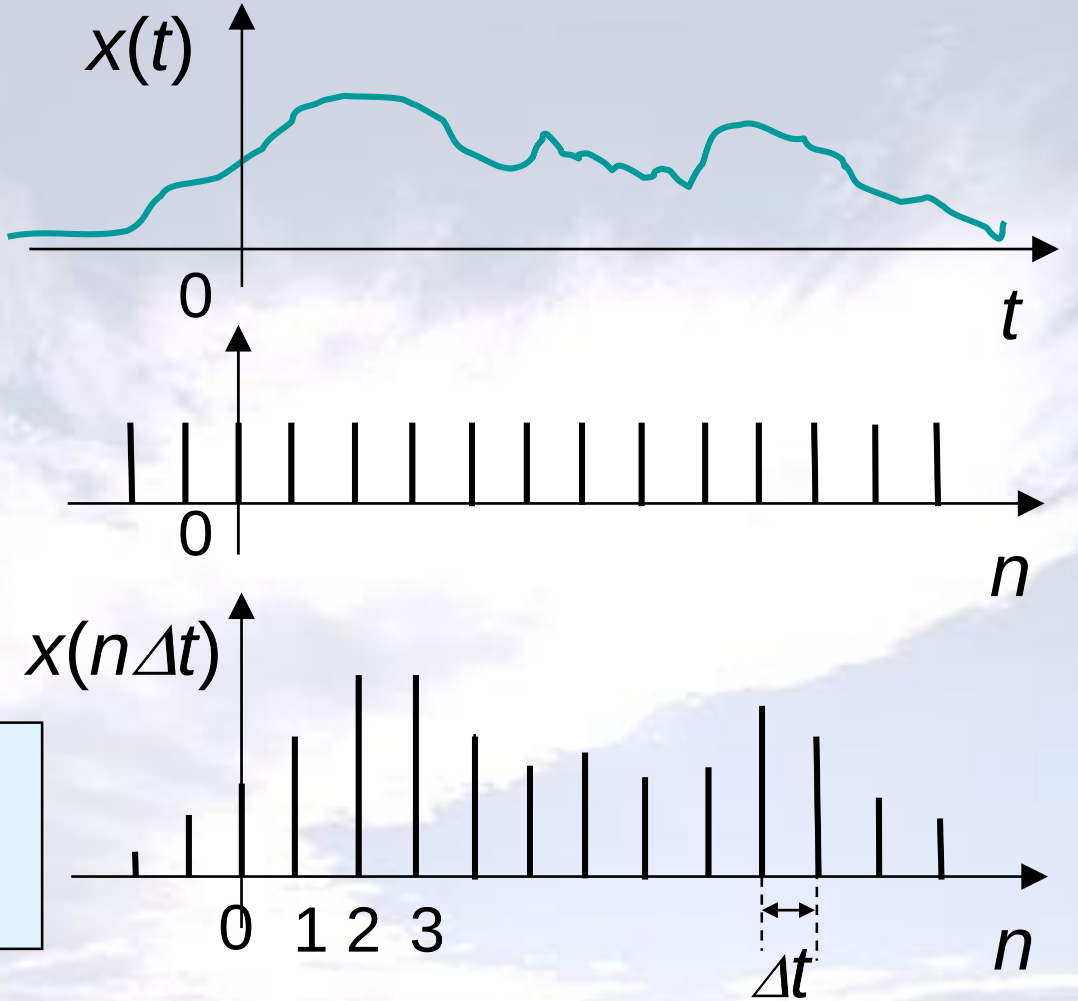
$$x(t)$$

x

$$\sum_{k=-\infty}^{k=\infty} \delta(n-k)$$

=

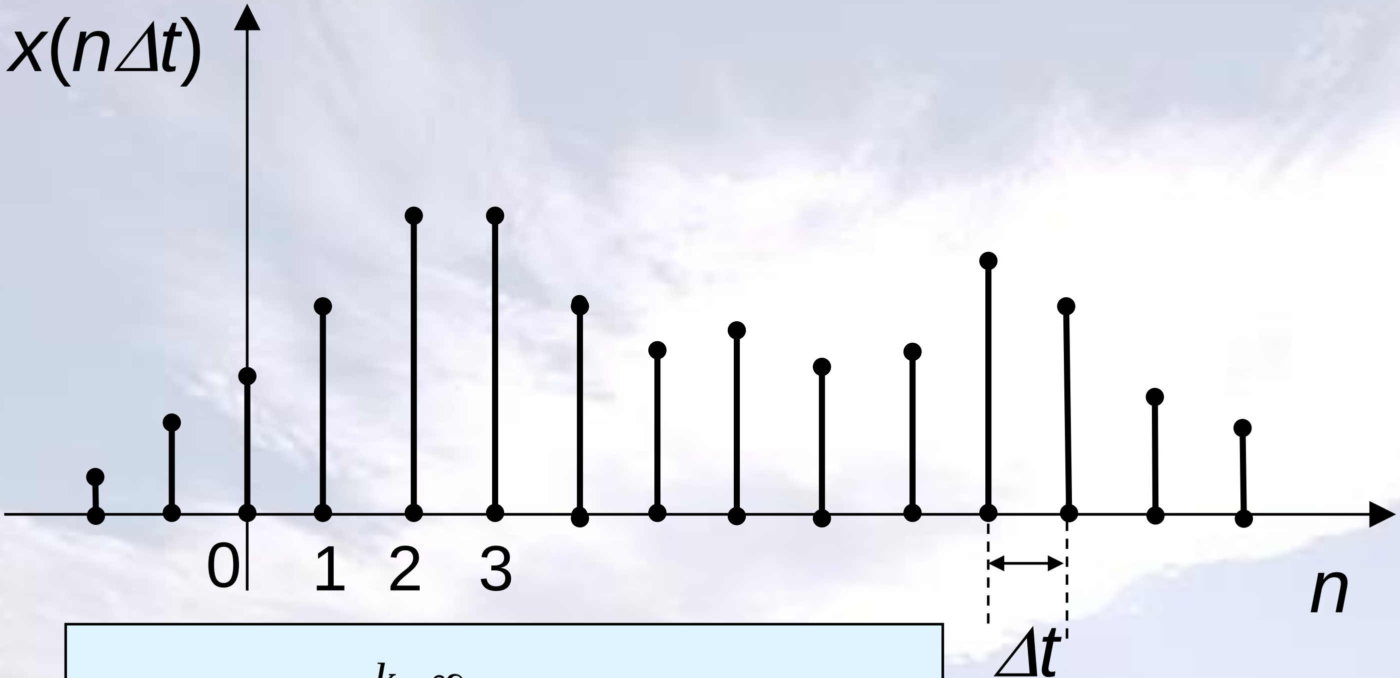
$$x(n) = \sum_{k=-\infty}^{k=\infty} x(t)\delta(n-k)$$



Discrete signal definition

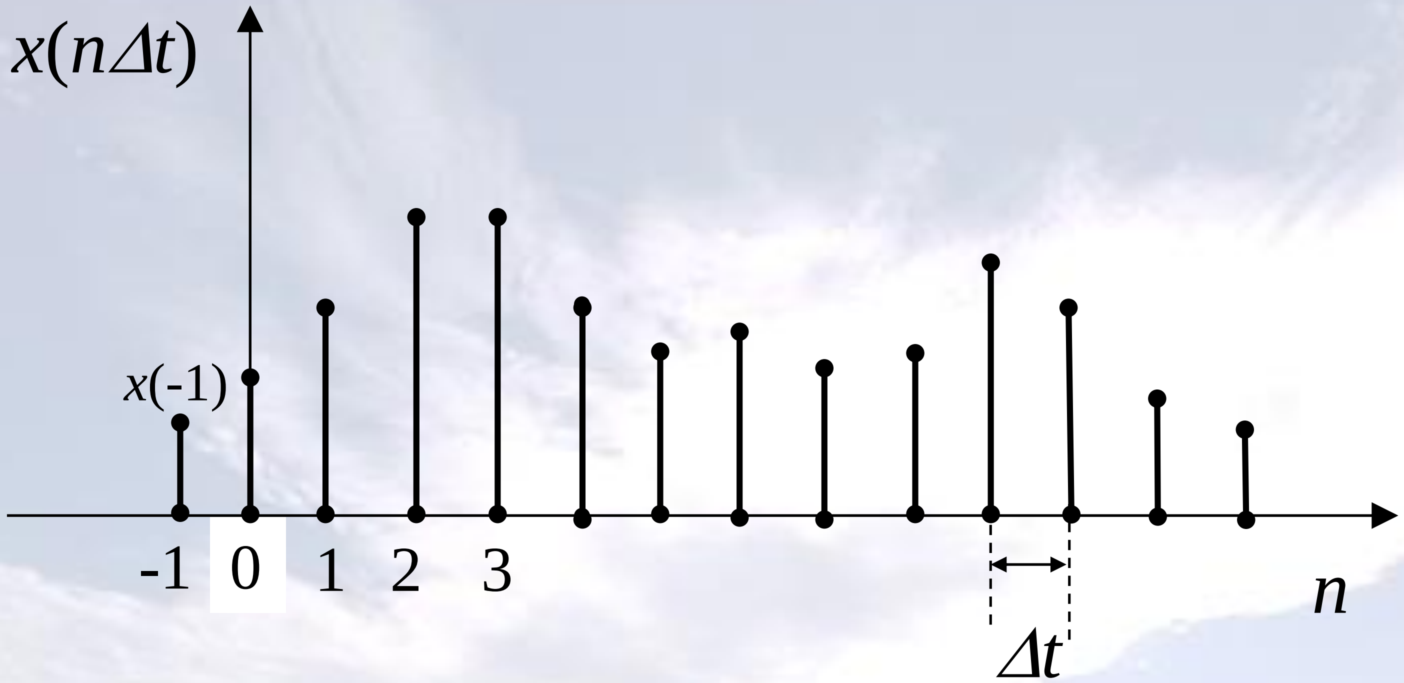
Discrete time signal is an impulse series: $\{x(n\Delta t)\}$

for $\Delta t=1$ $\{x(1), x(2), \dots x(k), \dots\}$



$$x(n)=\sum_{k=-\infty}^{k=\infty} x(k)\delta(n-k)$$

Discrete signal example



$$x(n)=x(-1)\delta(n+1)+x(0)\delta(n-0)+x(1)\delta(n-1)+x(2)\delta(n-2)+\ldots$$

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Response of the LTI system to a discrete signal

Important

Input signal $x(n)$:

$$x(n) = \sum_{k=-\infty}^{k=\infty} x(k)\delta(n-k)$$

Impulse response:

$$h(n) = H[\delta(n)]$$

The response of the system to the input $x(n)$ is determined according to the superposition principle:

$$y(n) = H\left[\sum_{k=-\infty}^{k=\infty} x(k)\delta(n-k)\right] = \sum_{k=-\infty}^{k=\infty} H[x(k)\delta(n-k)] =$$

$$= \sum_{k=-\infty}^{k=\infty} x(k)H[\delta(n-k)] = \sum_{k=-\infty}^{k=\infty} x(k)h(n-k)$$

Response of the LTI system to the discrete signal

Input signal $x(n)$:

$$\boxed{x(n)=\sum_{k=-\infty}^{k=\infty}x(k)\delta(n-k)}$$

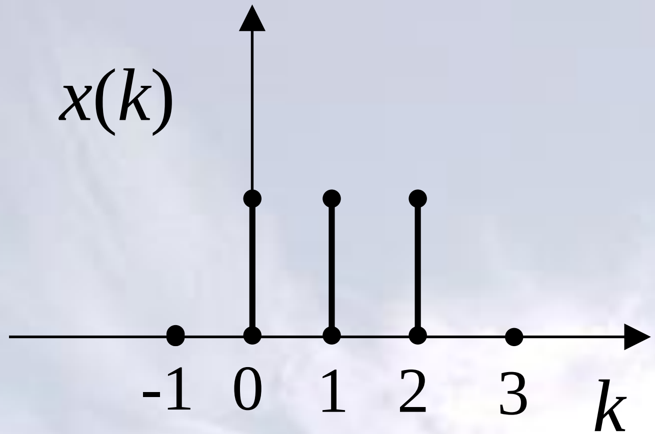
Output signal $y(n)$:

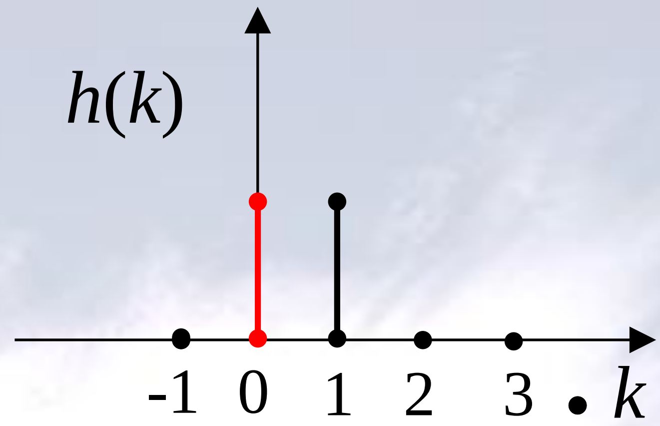
$$\boxed{y(n)=\sum_{k=-\infty}^{k=\infty}x(k)h(n-k)=\sum_{k=-\infty}^{k=\infty}x(n-k)h(k)}$$

Convolution example

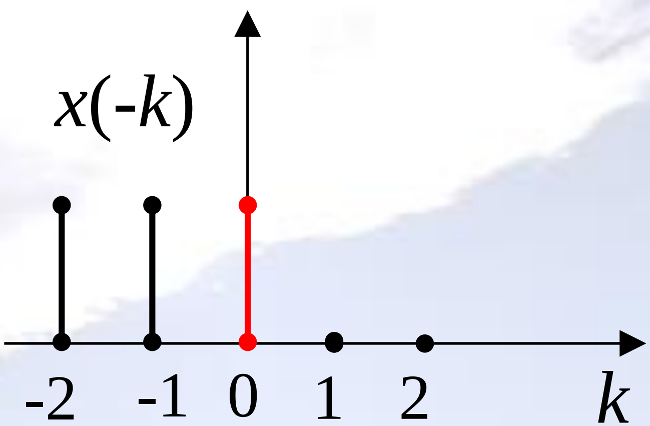
$$x(n)=[1,\ 1,\ 1]$$

$$h(n)=[1,\ 1]$$





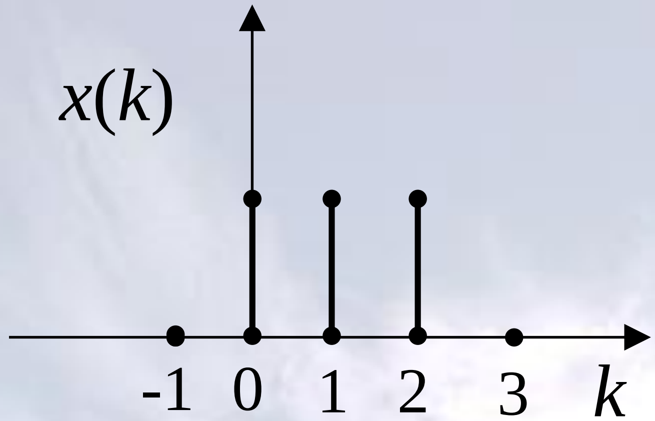
$$y(n)=\sum_{k=-\infty}^{k=\infty} x(n-k)h(k)$$

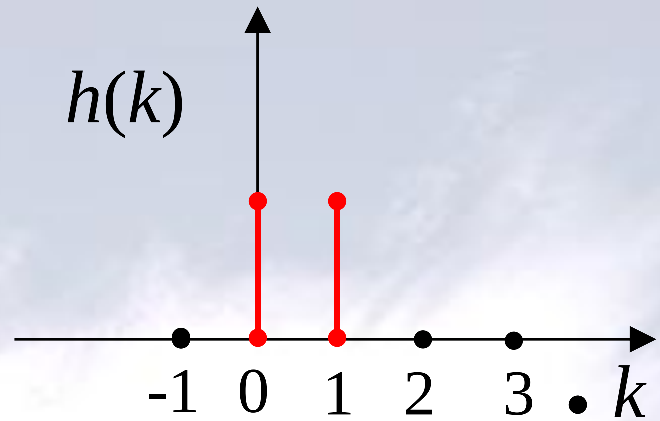


$$y(0)=\sum_{k=0}^{k=\infty} x(-k)h(k)=x(-0)h(0)=1$$

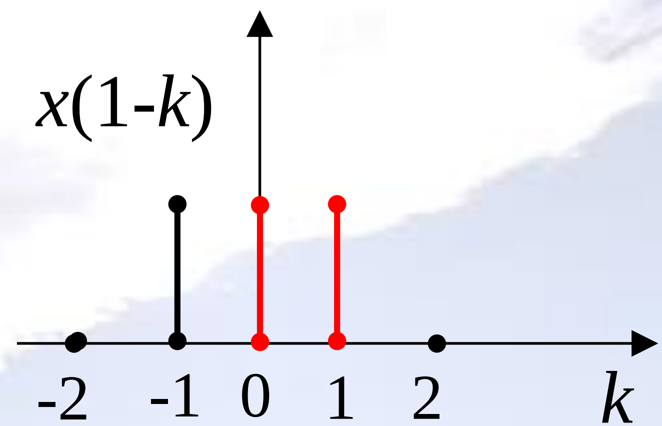
Convolution example

$$x(n)=[1,\ 1,\ 1]$$
$$h(n)=[1,\ 1]$$





$$\boxed{y(n)=\sum_{k=-\infty}^{k=\infty}x(n-k)h(k)}$$



$$y(1)=\sum_{k=0}^{k=\infty}x(1-k)h(k)=x(1-0)h(0)+x(1-1)h(1)+$$
$$+x(1-2)h(2)+\ldots=1+1+0=2$$

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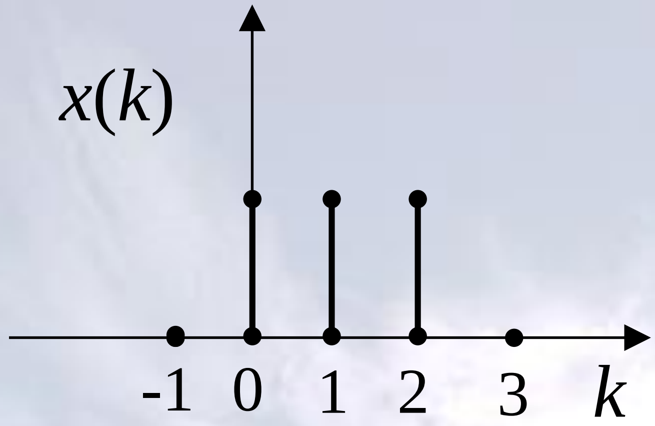
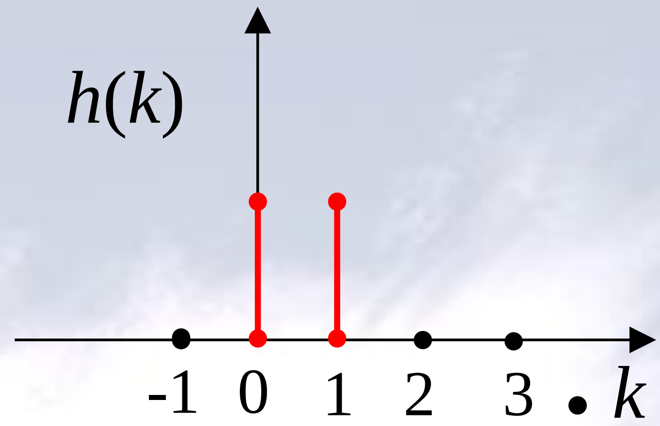
Convolution example

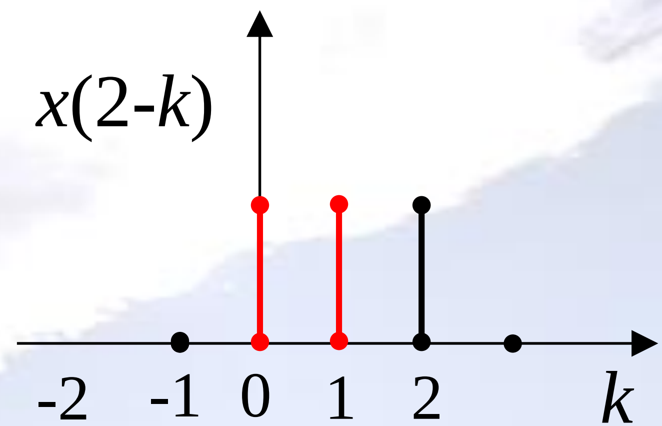
$$x(n) = [1,\ 1,\ 1]$$
$$h(n) = [1,\ 1]$$





$$y(n) = \sum_{k=-\infty}^{k=\infty} x(n-k)h(k)$$

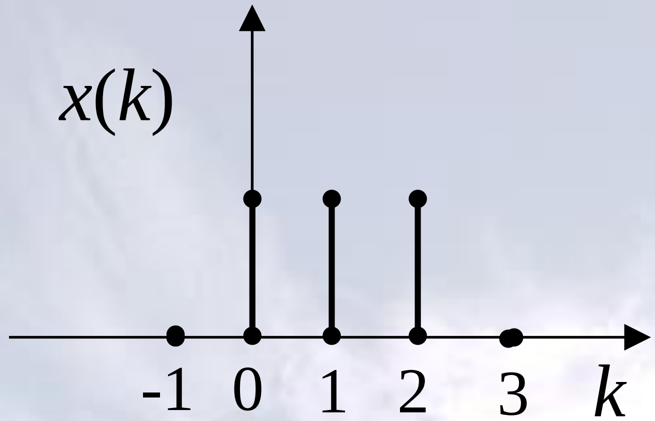
$$y(2) = \sum_{k=0}^{k=\infty} x(2-k)h(k) = x(2-0)h(0) + x(2-1)h(1) +$$
$$+\, x(2-2)h(2) + \ldots = 1 + 1 + 0 = 2$$

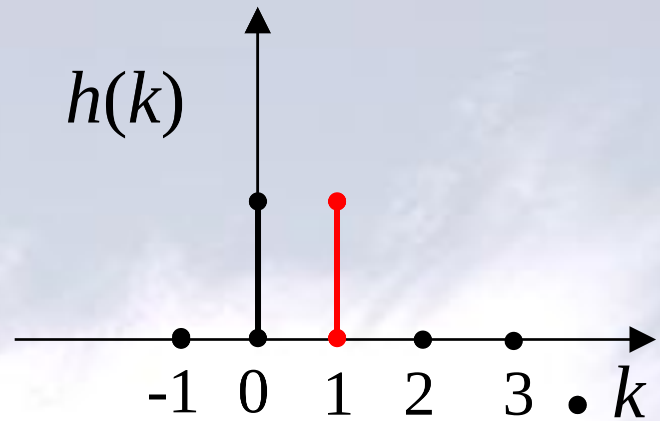


Convolution example

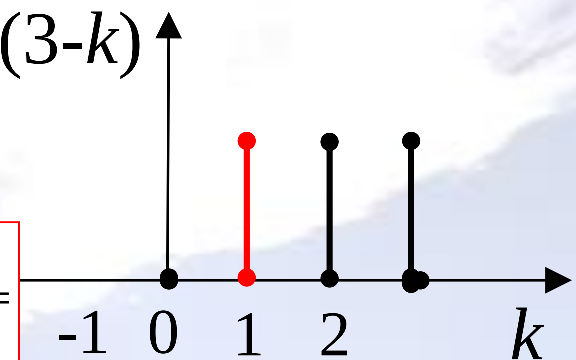
$$x(n) = [1,\ 1,\ 1]$$

$$h(n) = [1,\ 1]$$





$$y(n) = \sum_{k=-\infty}^{k=\infty} x(n-k)h(k)$$



$$y(3) = \sum_{k=0}^{k=\infty} x(3-k)h(k) = x(3-0)h(0) + x(3-1)h(1) + x(3-2)h(2) =$$

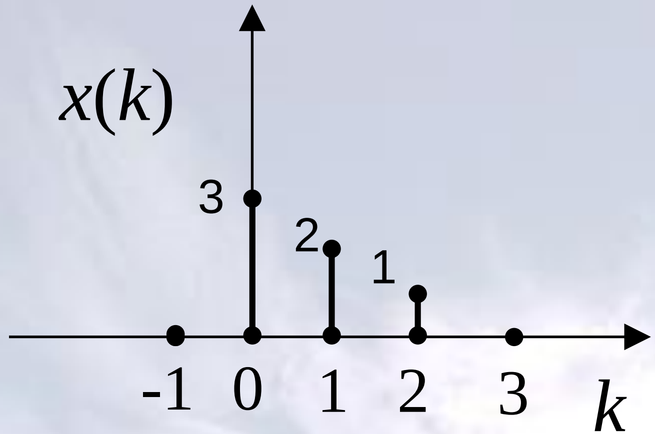
$$= 0\cdot 1 + 1\cdot 1 + 1\cdot 0 + 1\cdot 0 = 1$$

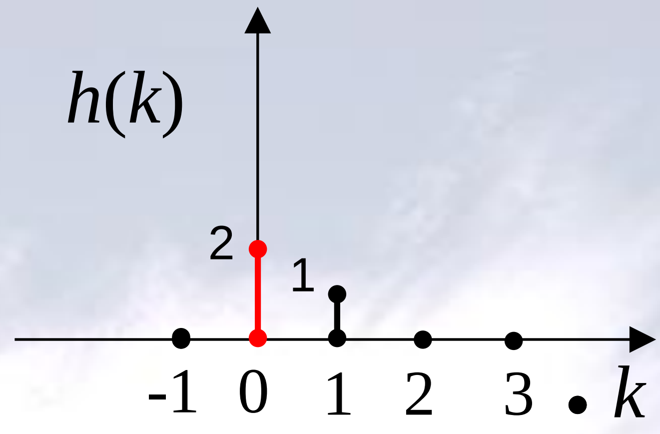
$$y(n) = x(n) * h(n) = [1\ 2\ 2\ 1]$$

Convolution example

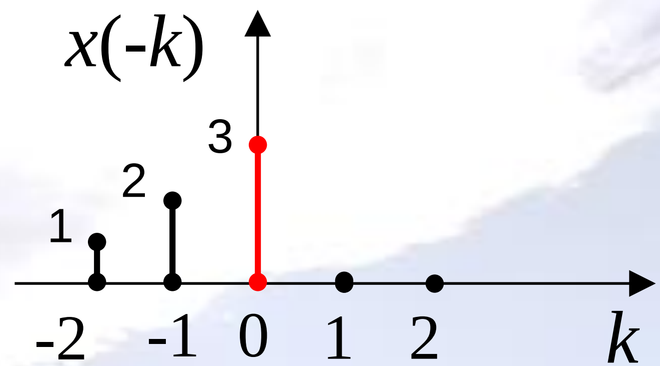
$$x(n) = [3,2,1]$$

$$h(n) = [2,1]$$





$$y(n) = \sum_{k=-\infty}^{k=\infty} x(n-k)h(k)$$

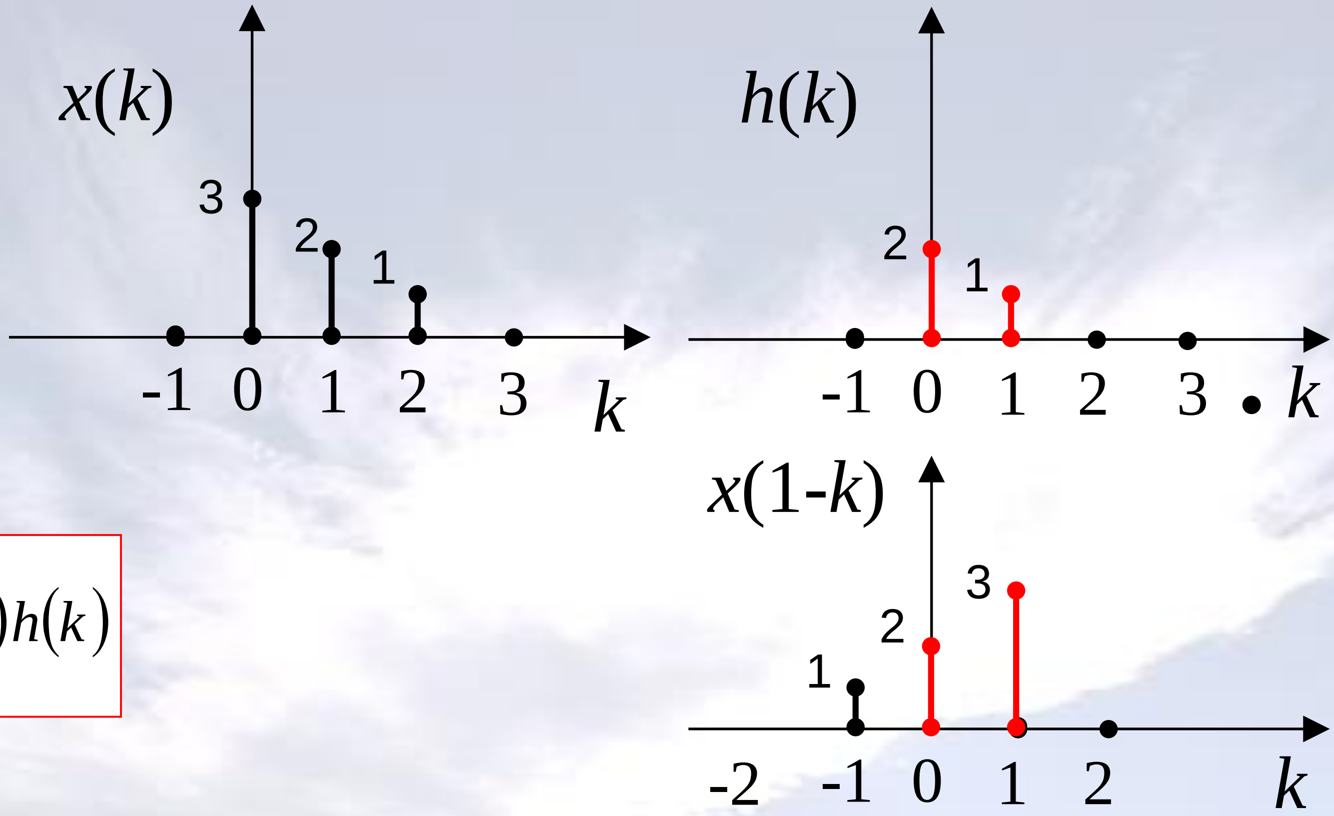


$$y(0) = \sum_{k=0}^{k=\infty} x(0-k)h(k) = x(-0)h(0) = 3 \cdot 2 = 6$$

Convolution example

$x(n) = [3,2,1]$

$h(n) = [2,1]$



$$y(n) = \sum_{k=-\infty}^{k=\infty} x(n-k)h(k)$$

$$y(1) = \sum_{k=0}^{k=\infty} x(1-k)h(k) = x(1-0)h(0) + x(1-1)h(1) + x(1-2)h(2) = 4 + 3 + 0 = 7$$

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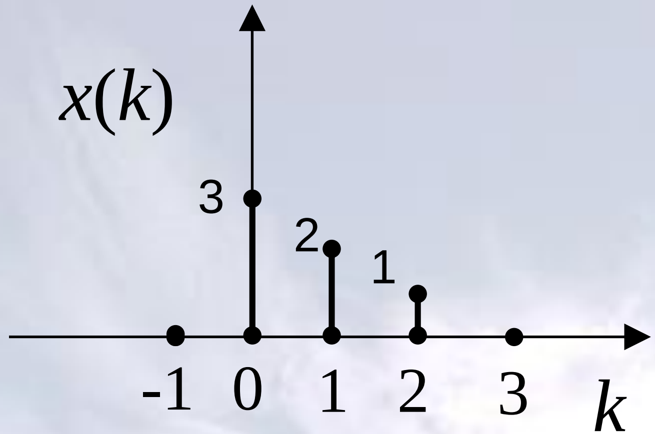
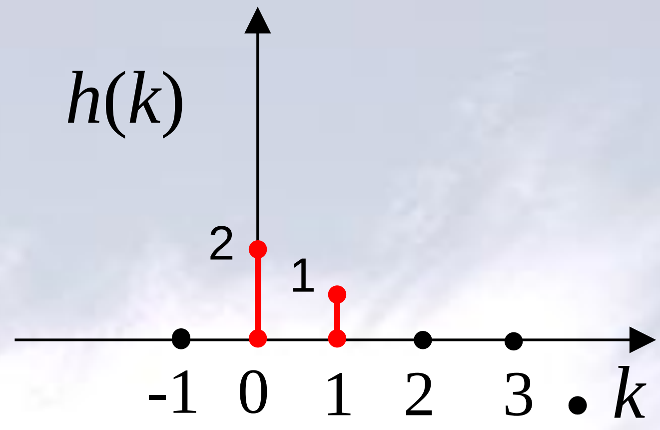
Convolution example

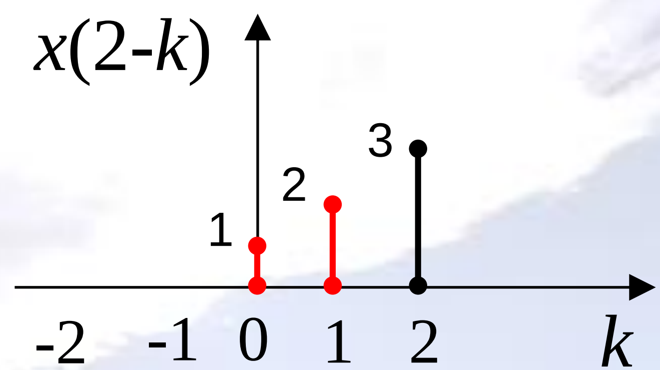
$$x(n) = [3,2,1]$$

$$h(n) = [2,1]$$





$$y(n) = \sum_{k=-\infty}^{k=\infty} x(n-k)h(k)$$

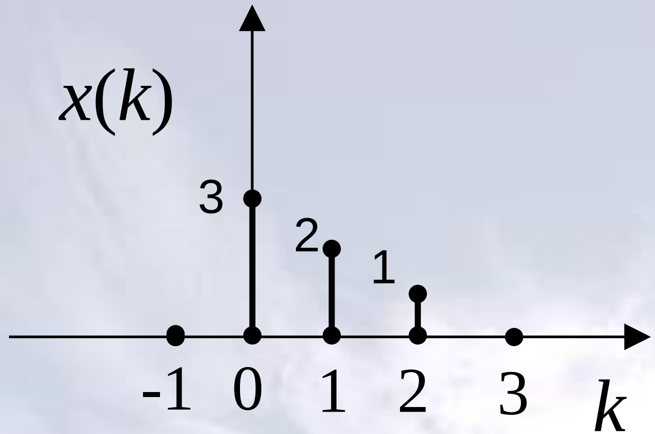


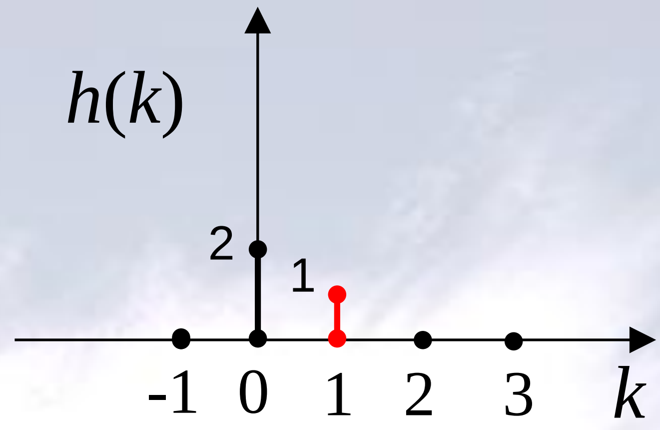
$$y(2) = \sum_{k=0}^{k=\infty} x(2-k)h(k) = x(2-0)h(0) + x(2-1)h(1) + x(2-2)h(2) + \ \dots = 4$$

Convolution example

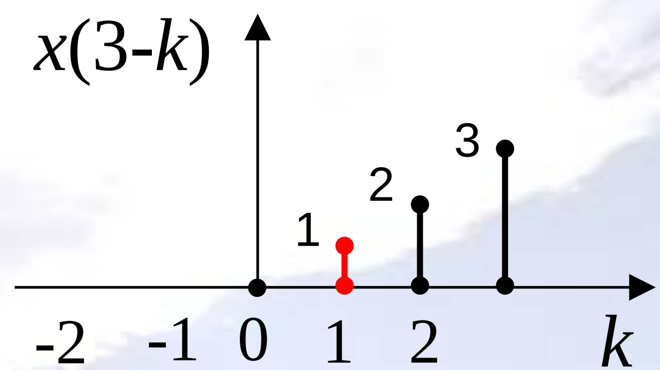
$$x(n) = [3,2,1]$$

$$h(n) = [2,1]$$



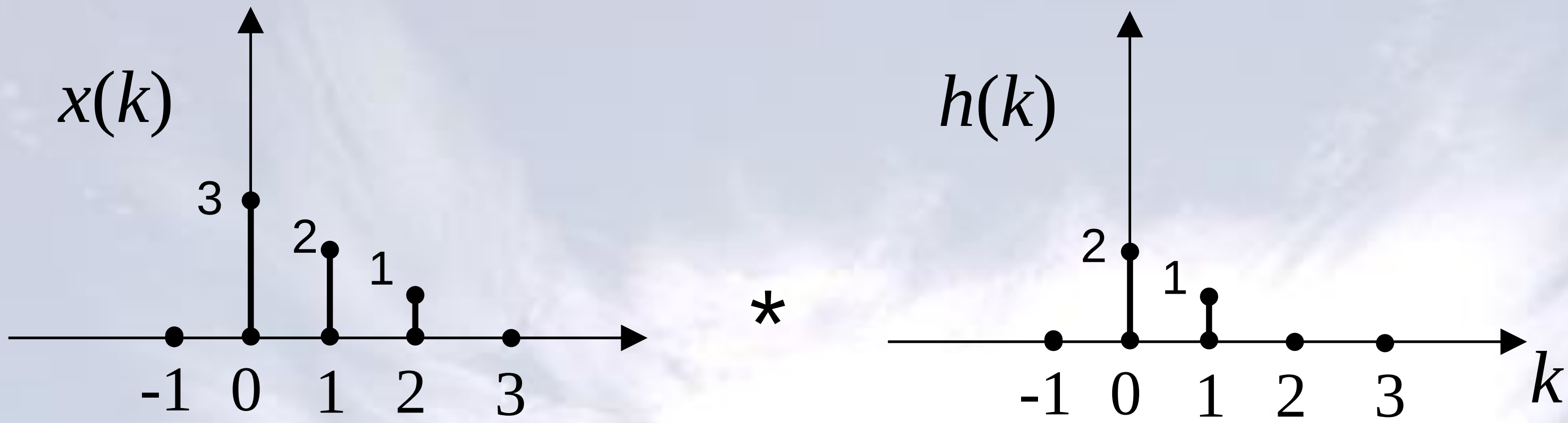


$$y(n) = \sum_{k=-\infty}^{k=\infty} x(n-k)h(k)$$



$$y(3) = \sum_{k=0}^{k=\infty} x(3-k)h(k) = x(3-0)h(0) + x(3-1)h(1) + x(3-2)h(2) = 1$$

Convolution example



$x(n) = [3,2,1]$

$h(n) = [2,1]$

$$y(n) = x(n) * h(n) = [3,2,1] * [2,1] = [6,7,4,1]$$

Convolution

Continuous:

$$y(t)=x(t)*h(t)=\int_{-\infty}^{+\infty}x(\tau)h(t-\tau)d\tau$$

Discrete:

$$y(n)=x(n)*h(n)=\sum_{k=-\infty}^{k=\infty}x(k)h(n-k)$$

Convolution

Equation:

$$\boxed{y(n)=\sum_{k=-\infty}^{k=\infty}x(k)h(n-k)=x(n)*h(n)}$$

is called the **convolution** of $x(n)$ with $h(n)$, ie. Input series with impulse response of the system.

$$\boxed{y(n)=\sum_{k=-\infty}^{k=\infty}x(k)h(n-k)=\sum_{k=-\infty}^{k=\infty}h(k)x(n-k)}$$

A response of a linear system to a periodic signal

Let the input signal be: $x(n) = e^{j\omega n}$

The output signal $y(n)$:

$$y(n) = \sum_{k=-\infty}^{k=\infty} x(n-k)h(k) = \sum_{k=-\infty}^{k=\infty} e^{j\omega(n-k)} h(k) =$$

$$= e^{j\omega n} \boxed{\sum_{k=-\infty}^{k=\infty} e^{-j\omega k} h(k)} = x(n)H(\omega)$$

$$e^{j\omega n} \longrightarrow \boxed{H(\omega)} \longrightarrow H(\omega)e^{j\omega n}$$

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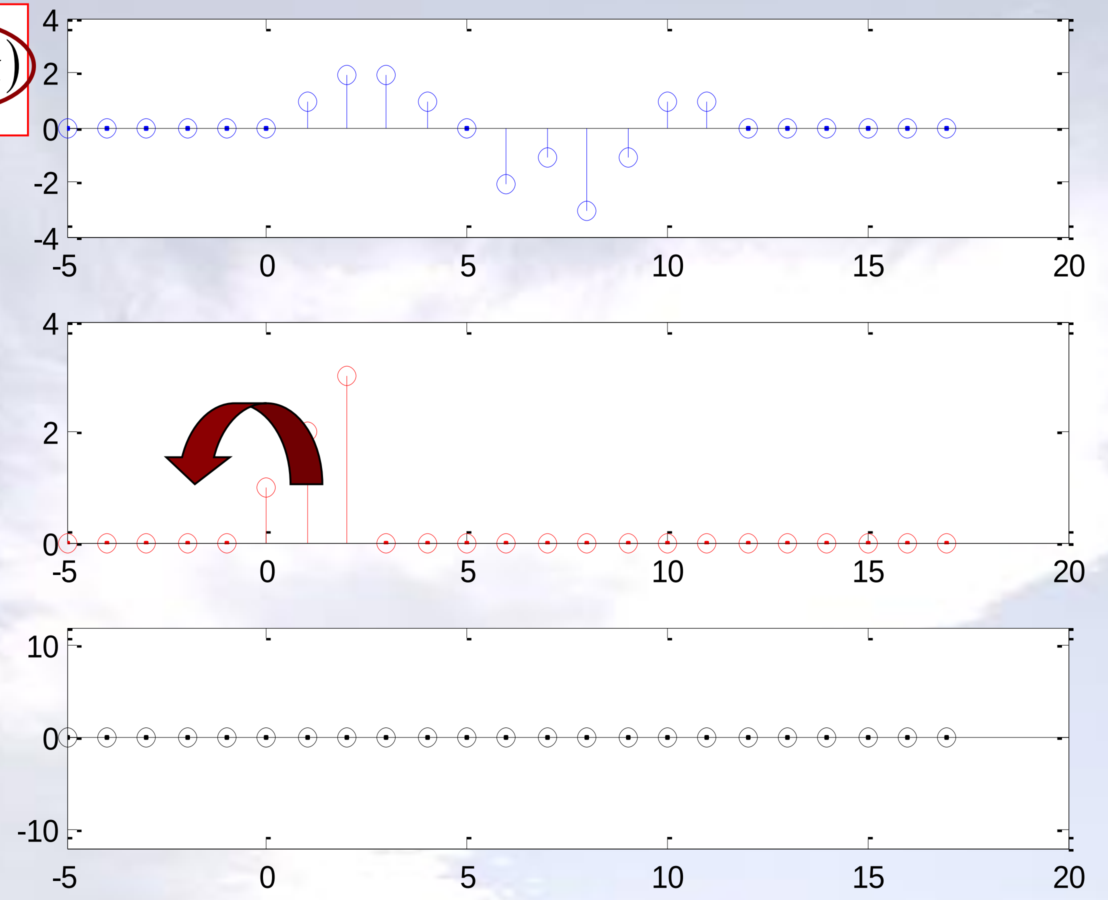
Determination of convolution

$$y(n) = x(n) * h(n) = \sum_{k=-\infty}^{k=\infty} x(k) h(n-k)$$

Signal *x*

Signal *h*

The result of convolution



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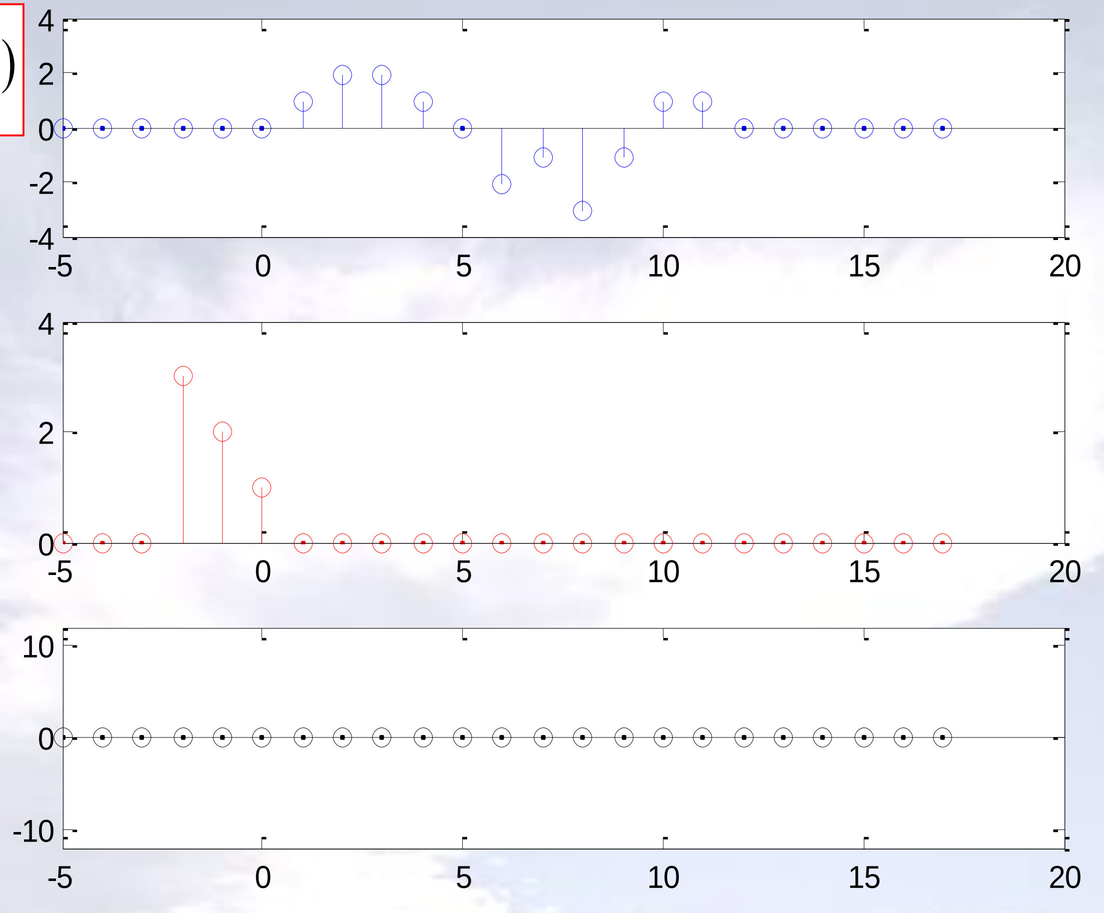

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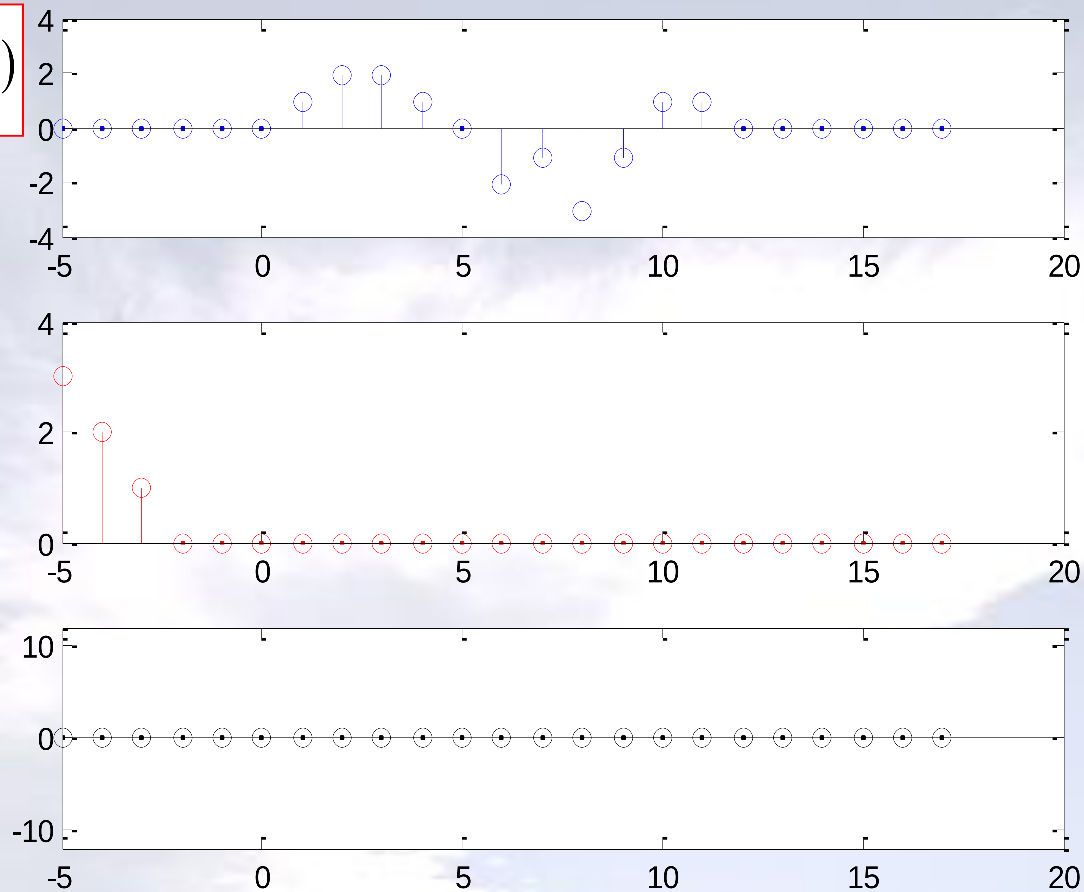

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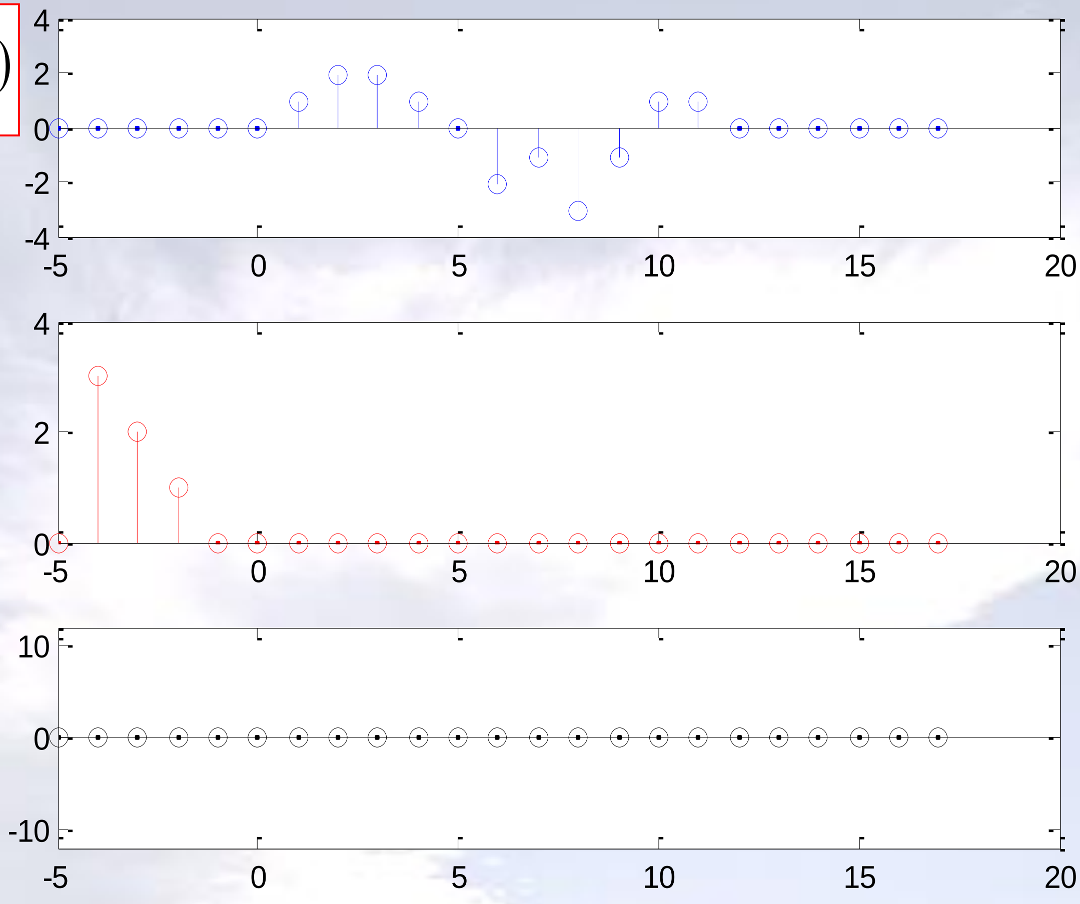

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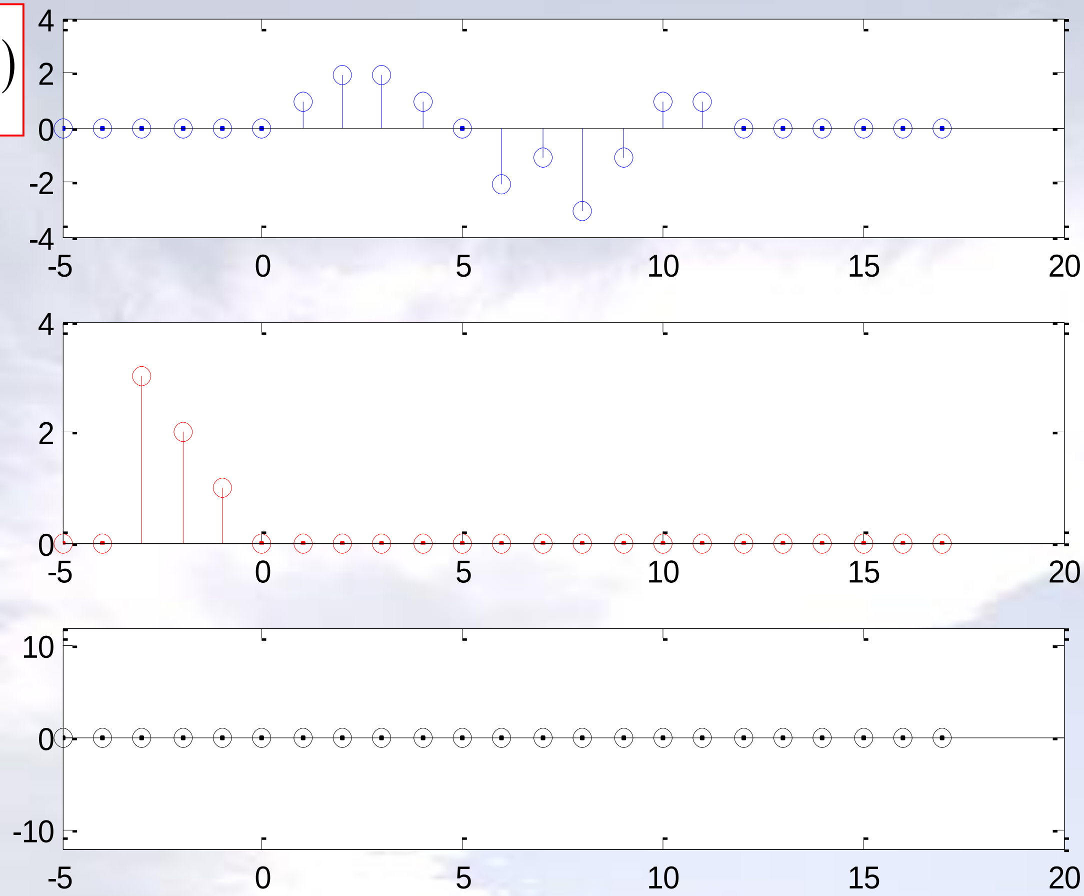

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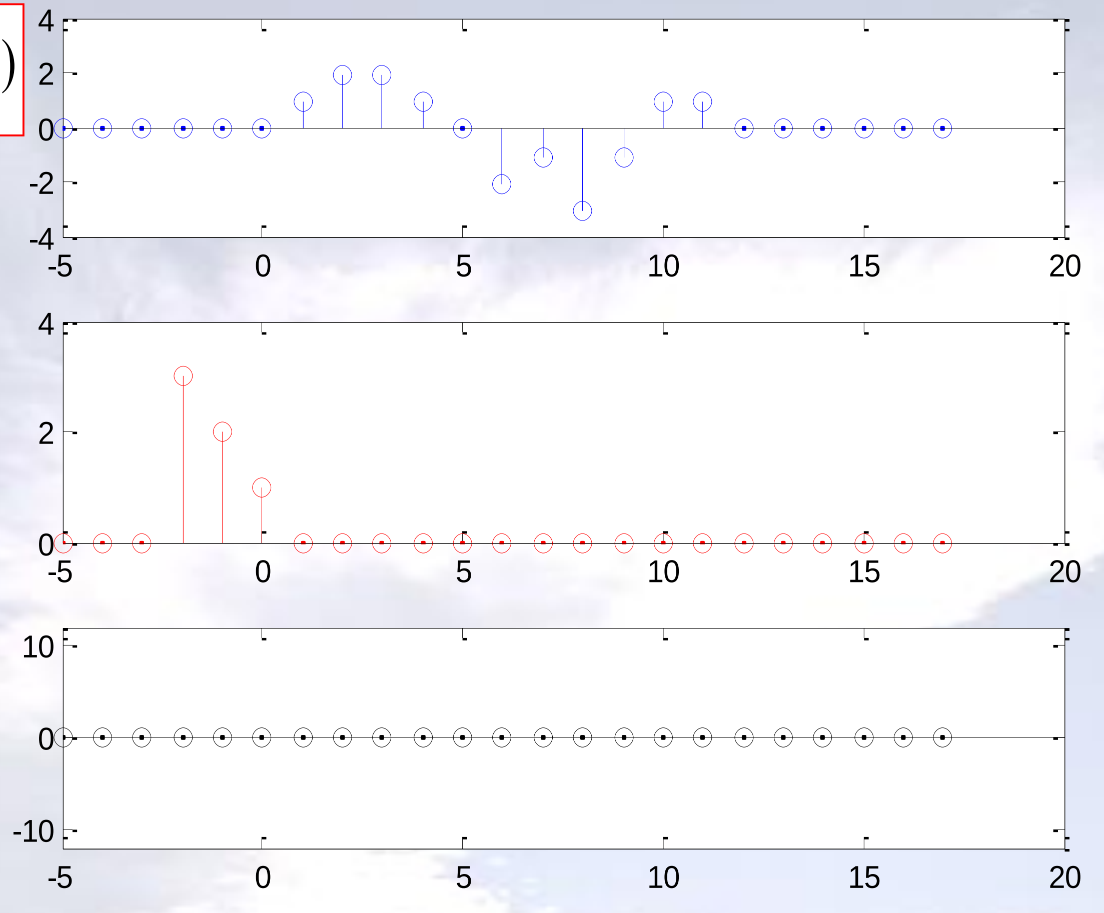

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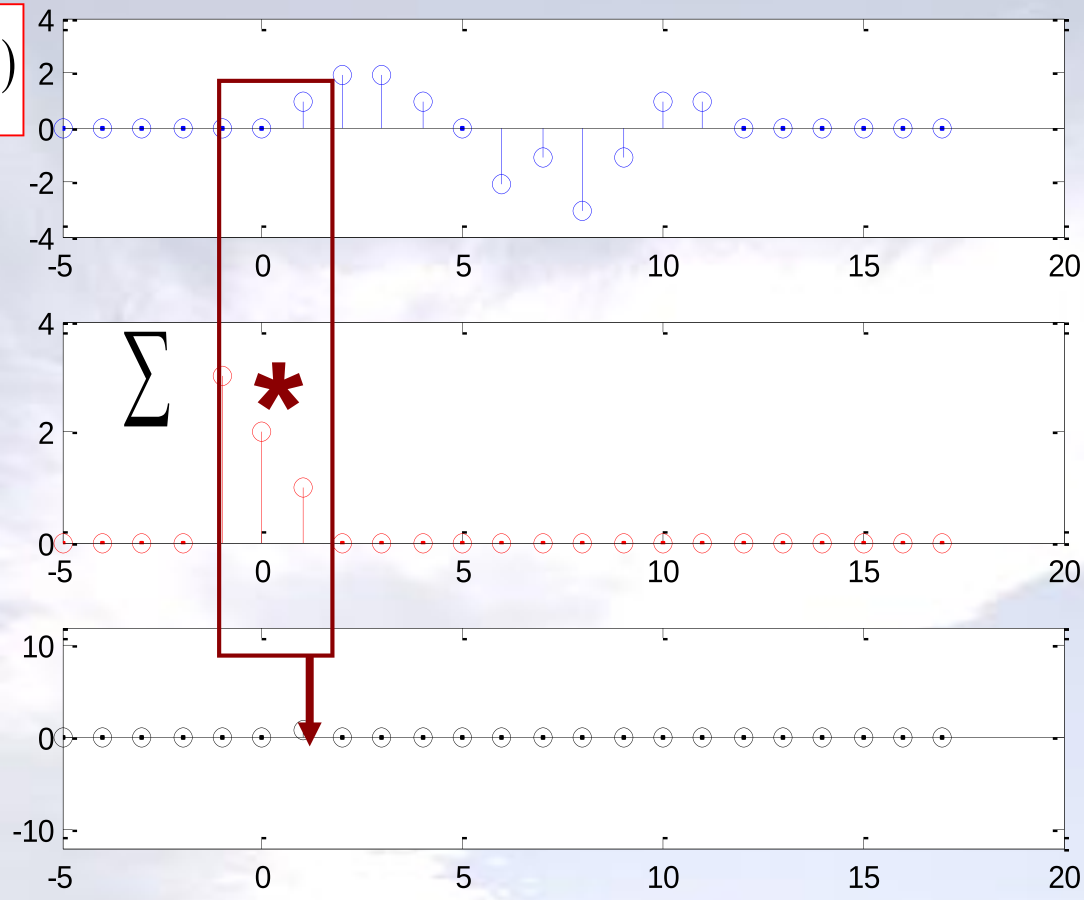

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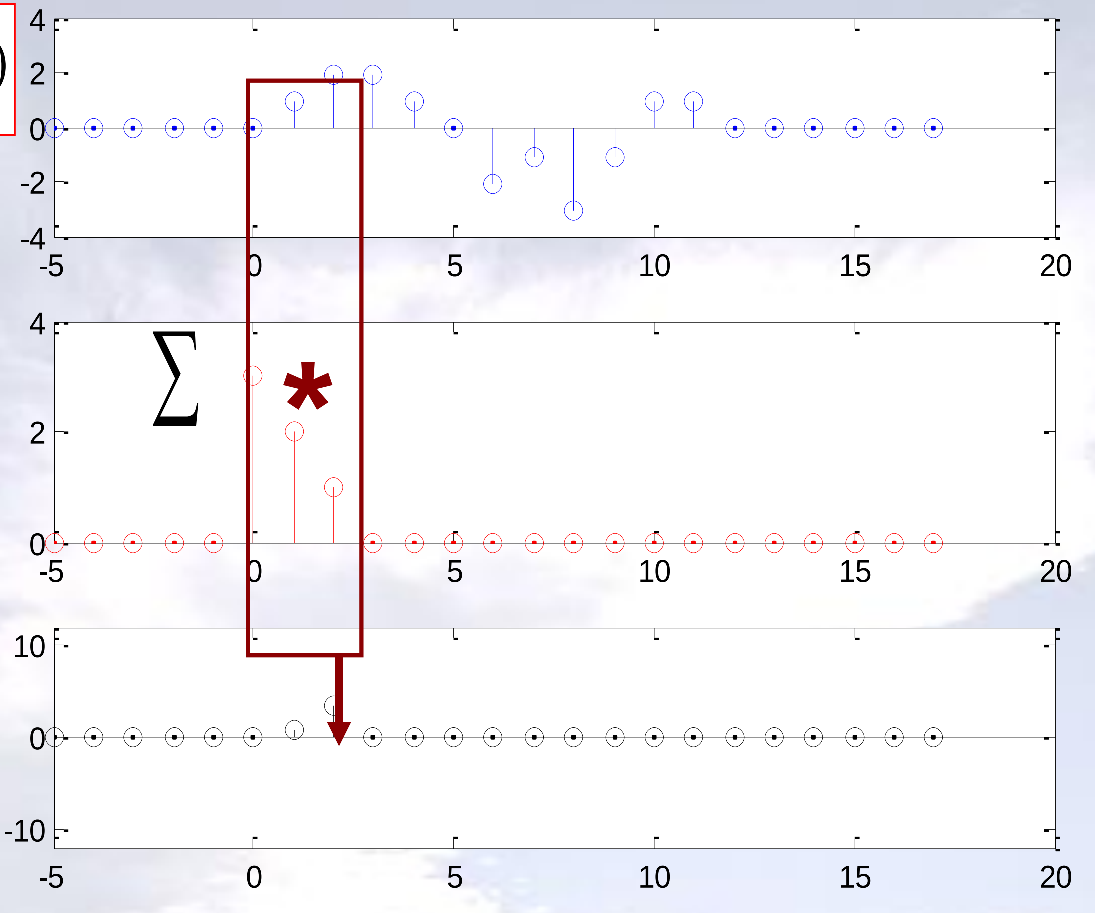

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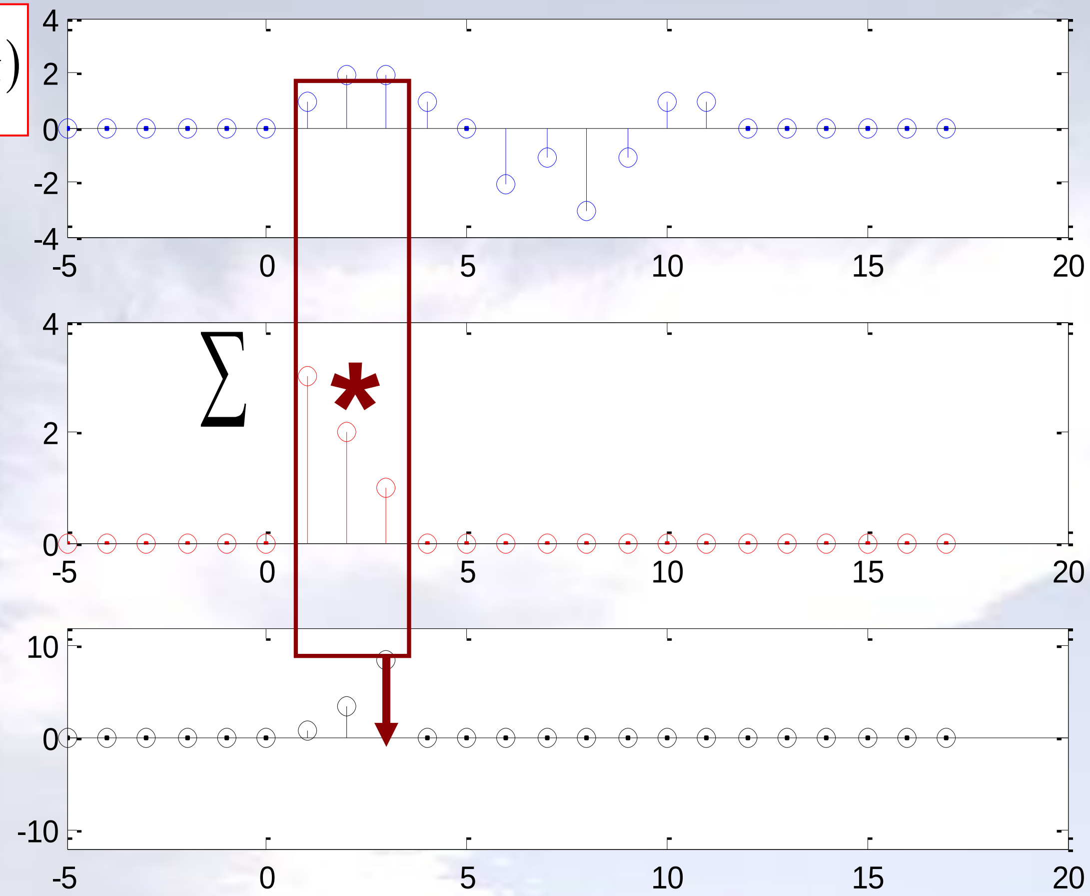

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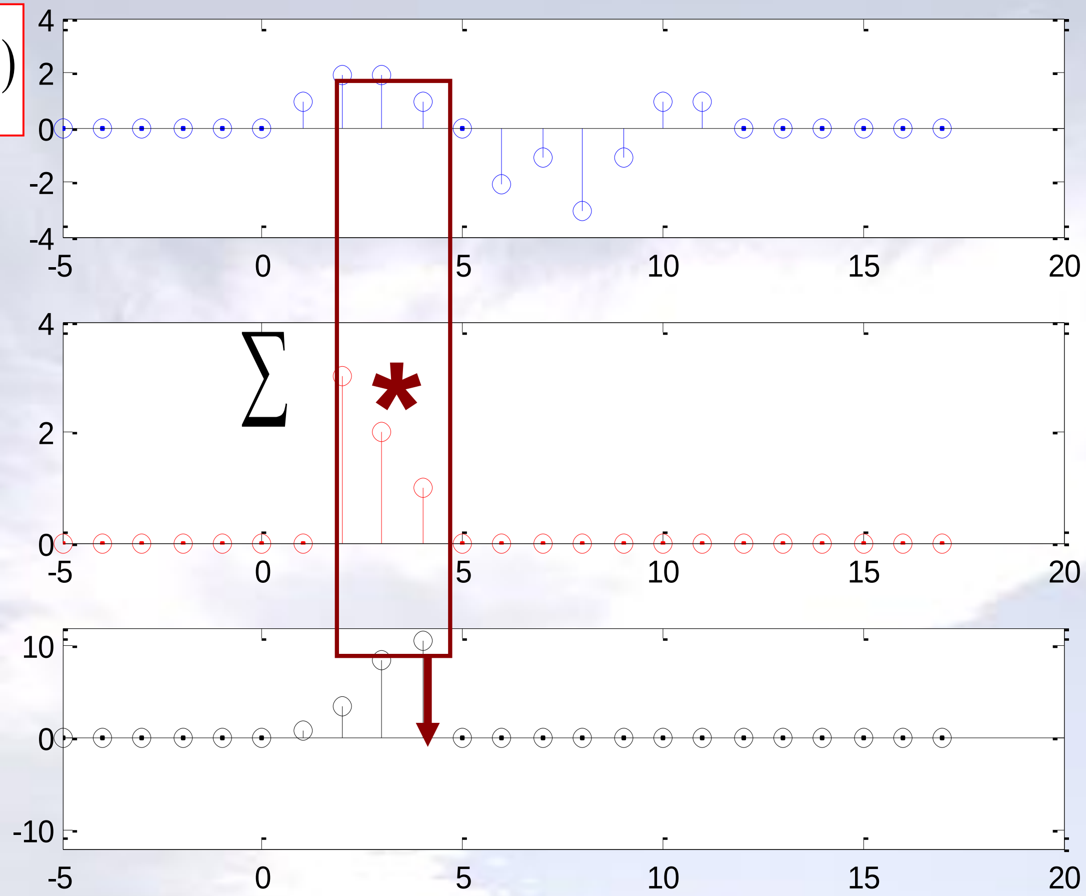

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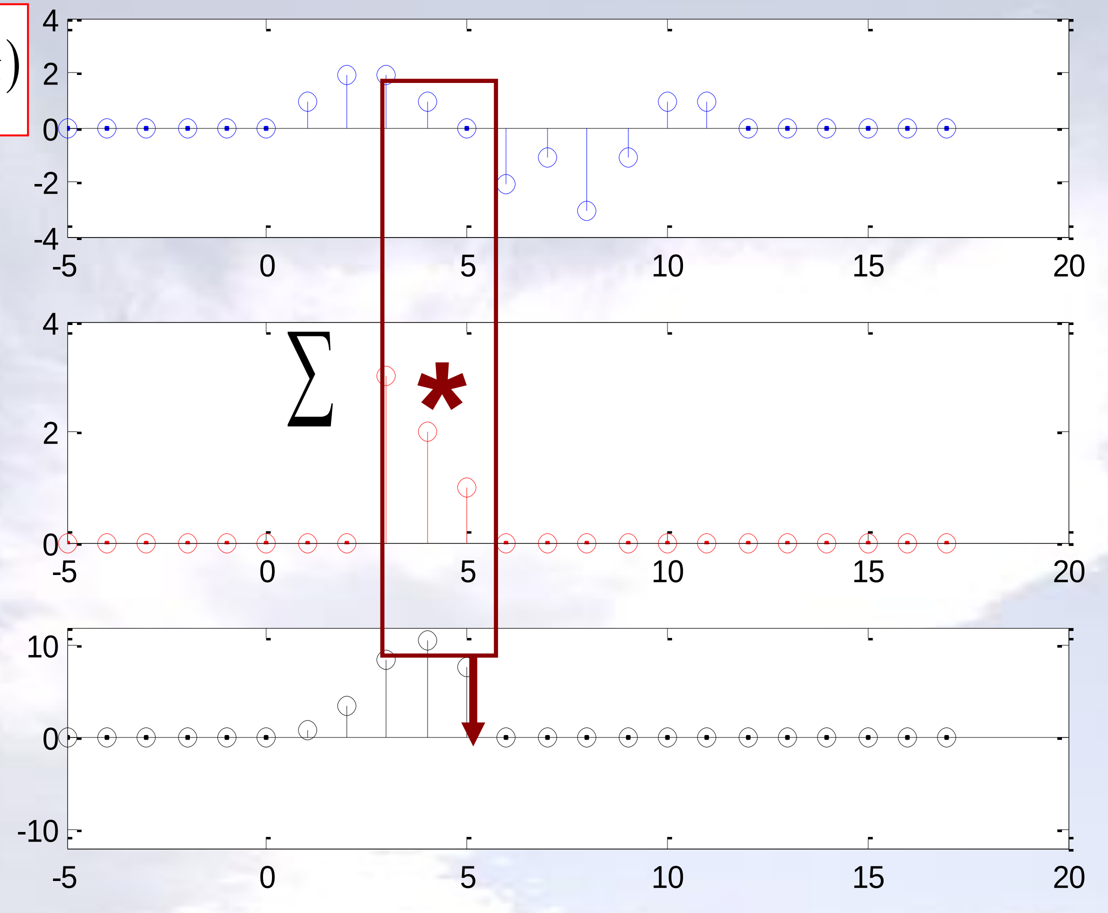

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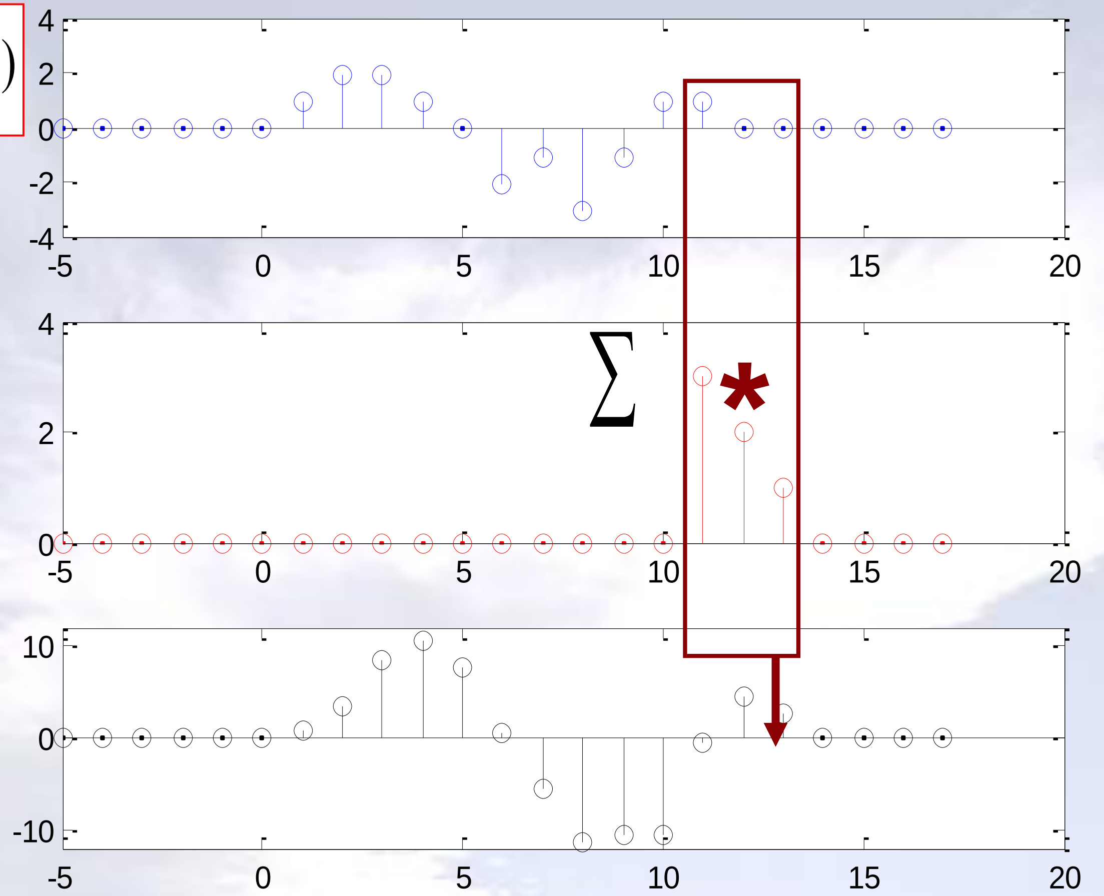

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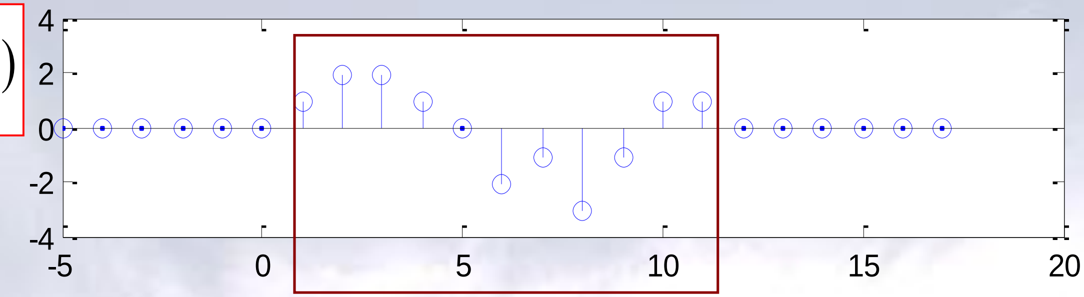
Signal *x*

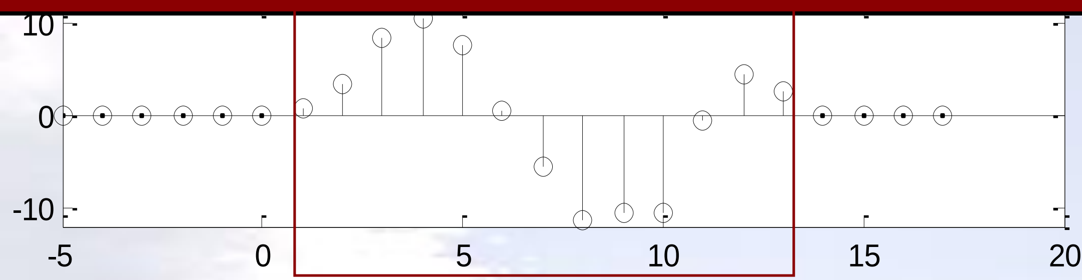


Pay attention to the length of the result.

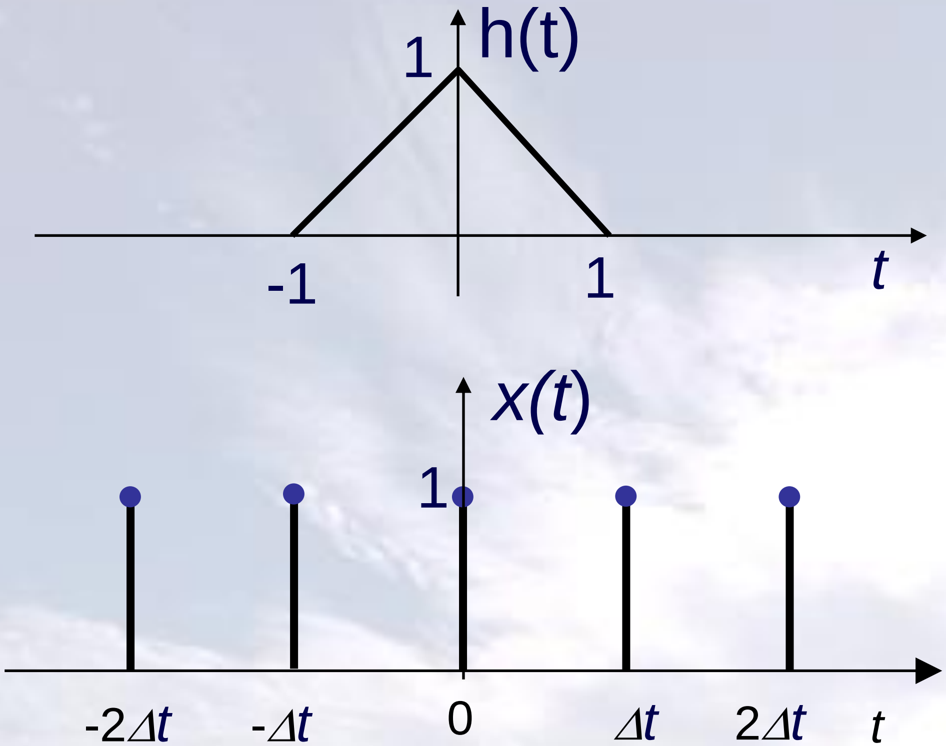
Compare the input signal *x* with its convolution with *h*.

The result of convolution

$$L = M + N - 1$$



Convolution - example



$$y(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau$$

Determine $y(t) = x(t) * h(t)$, for:

$\Delta t=4$, $\Delta t=2$, $\Delta t=3/2$ i $\Delta t=1$

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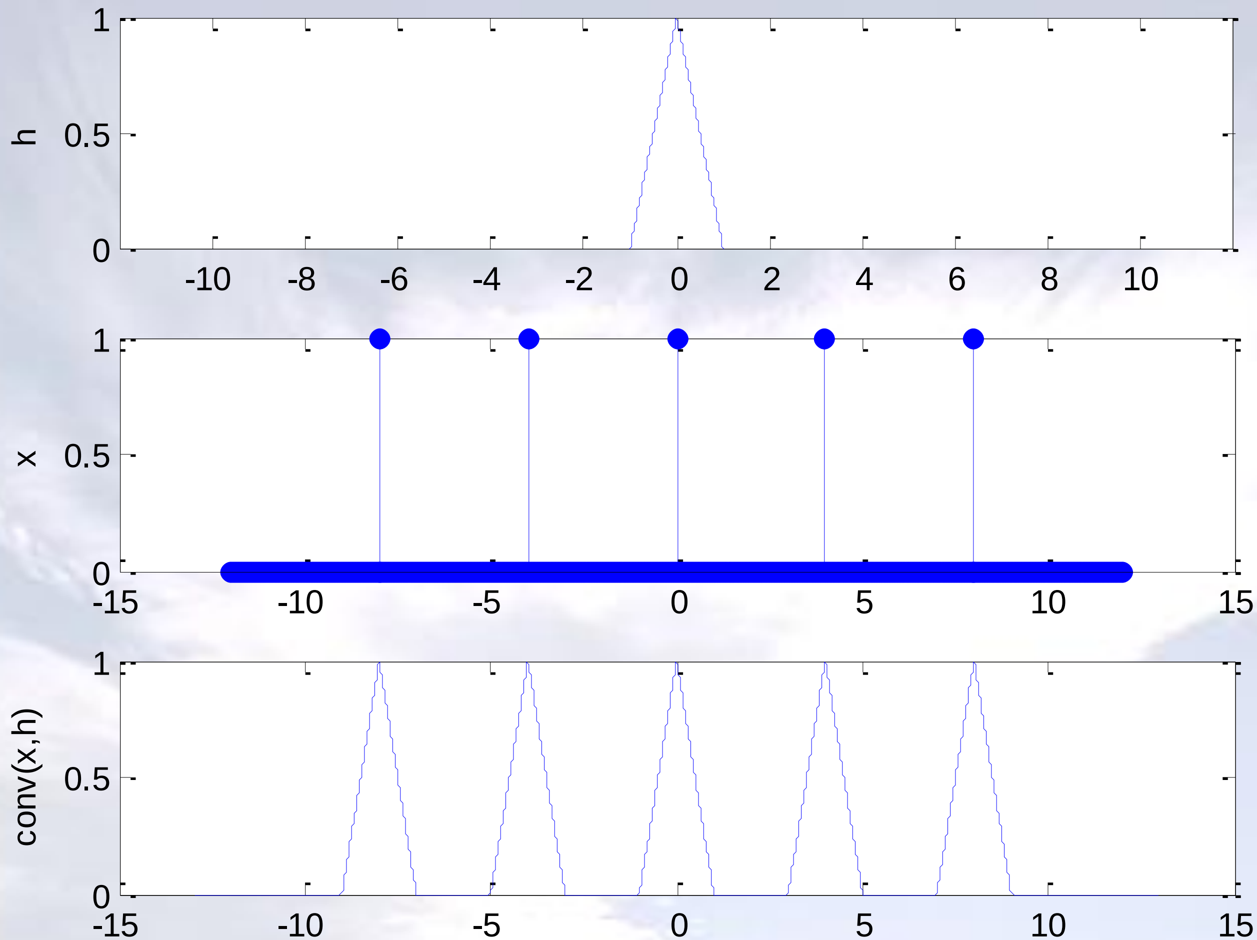
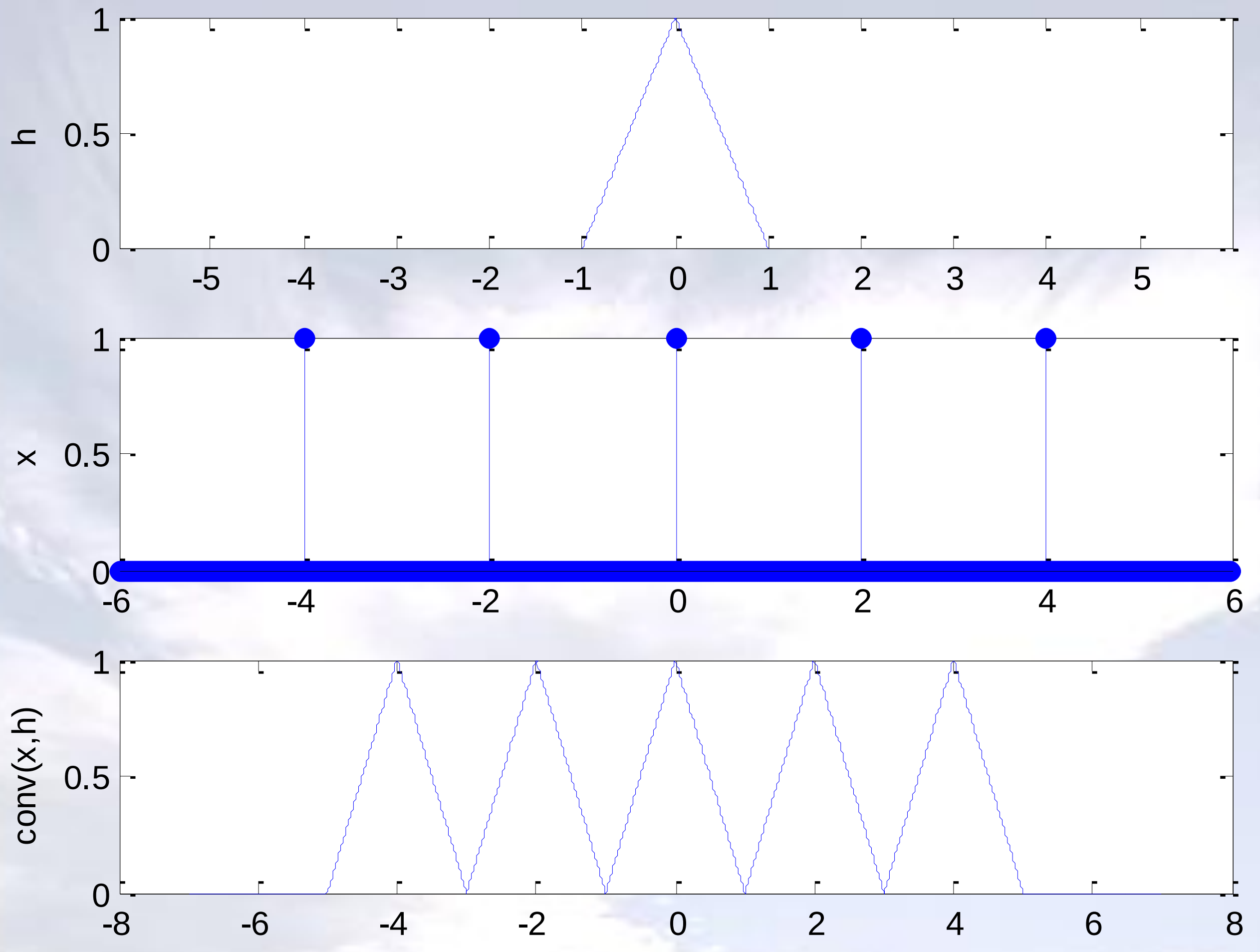
Convolution example

$\Delta t=4$



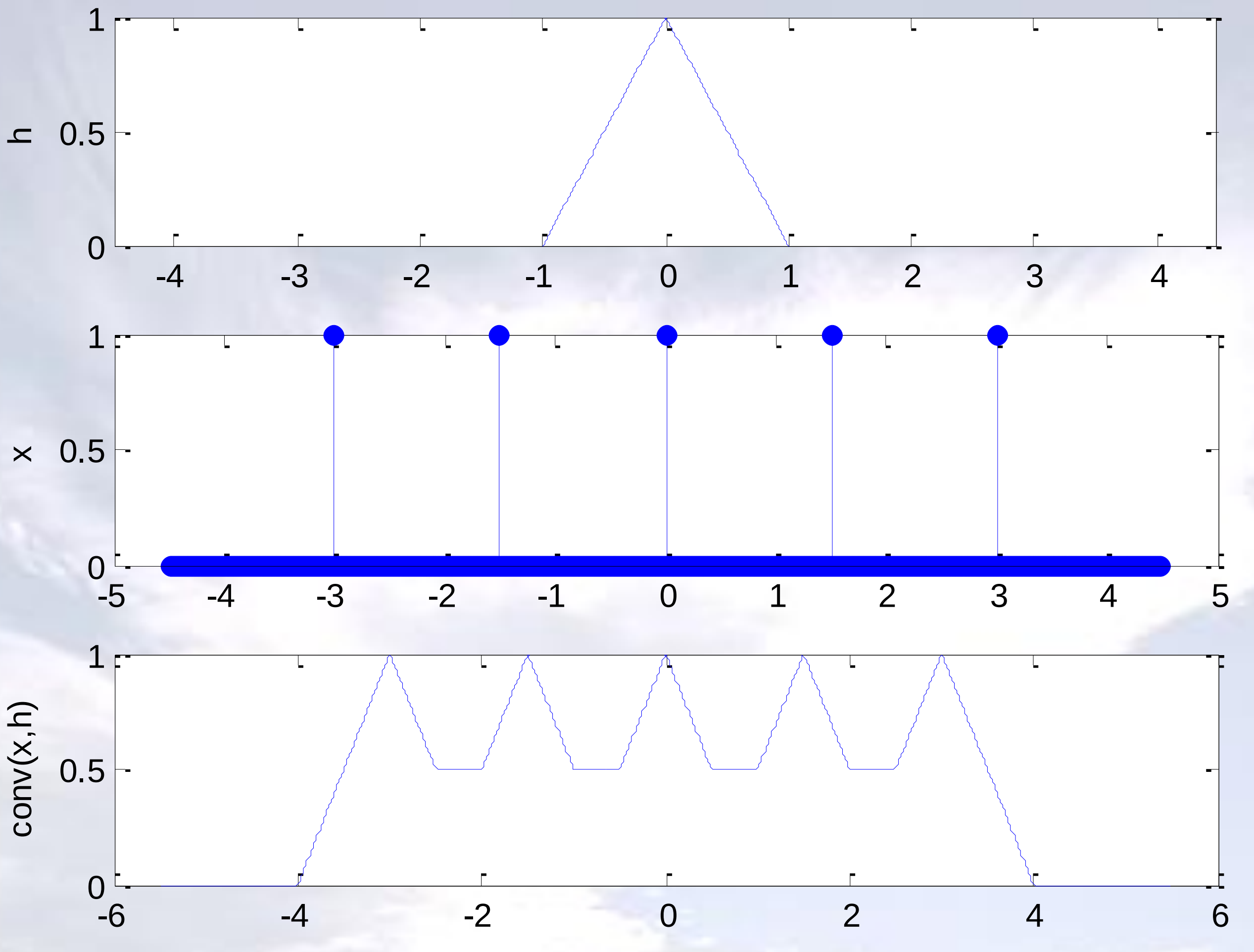
Convolution example

$\Delta t=2$



Convolution example

Δt=1.5



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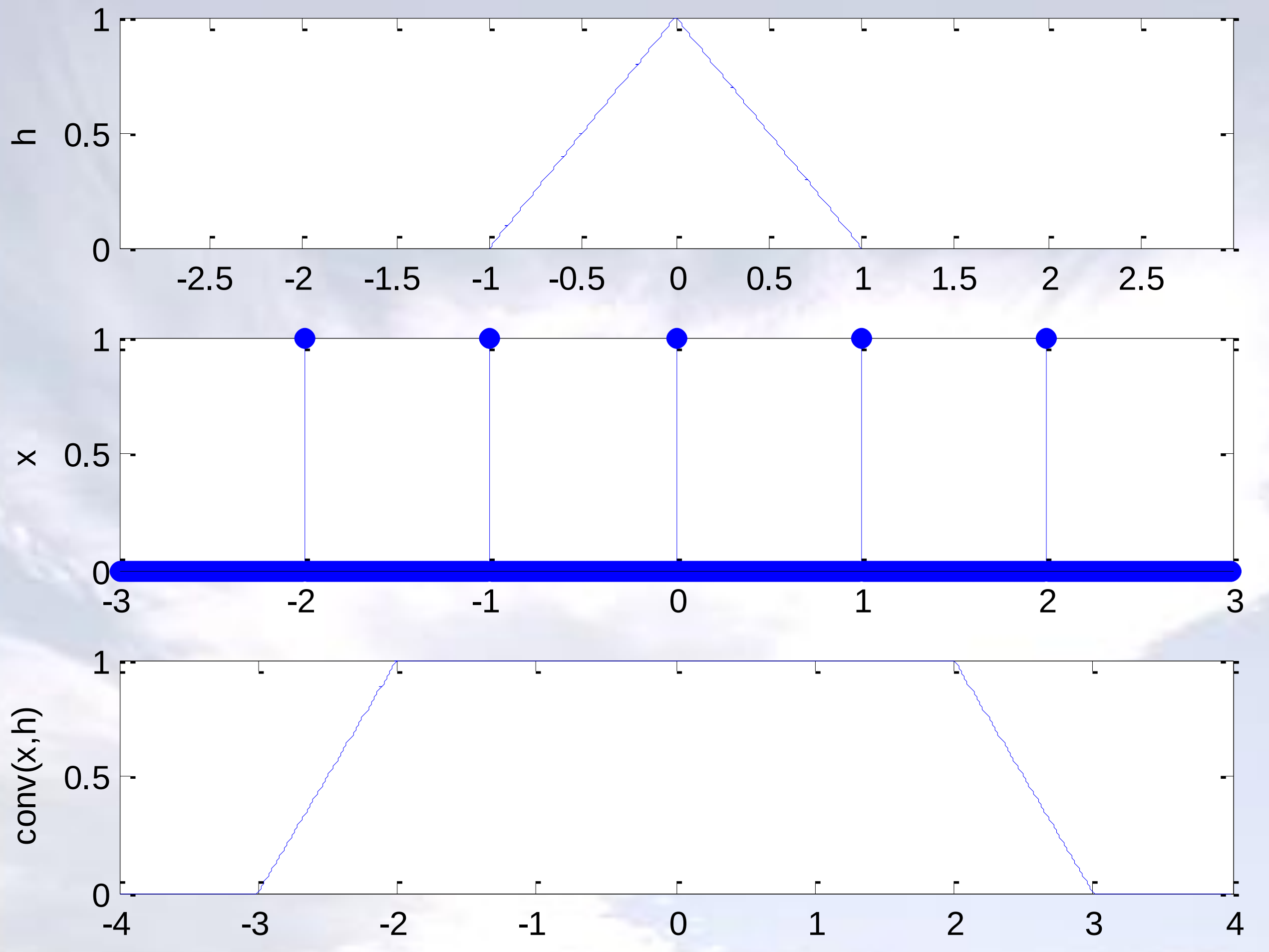
Convolution example

$\Delta t = 1$



Joy of Convolution

A free tutorial on convolution can be found in the internet at:

- http://www.jhu.edu/~signals/convolve/index.html

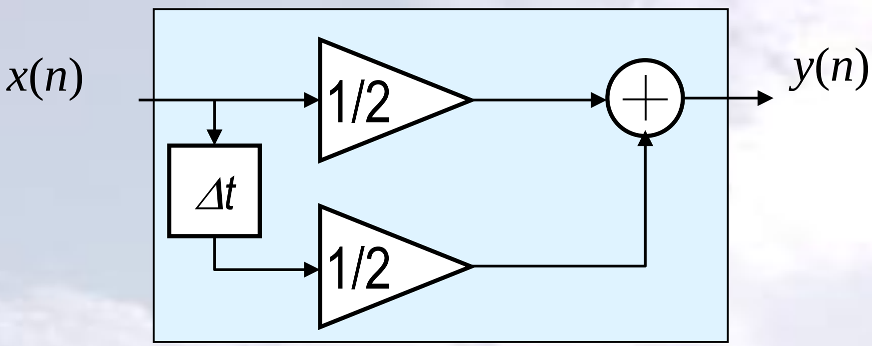
Convolution example

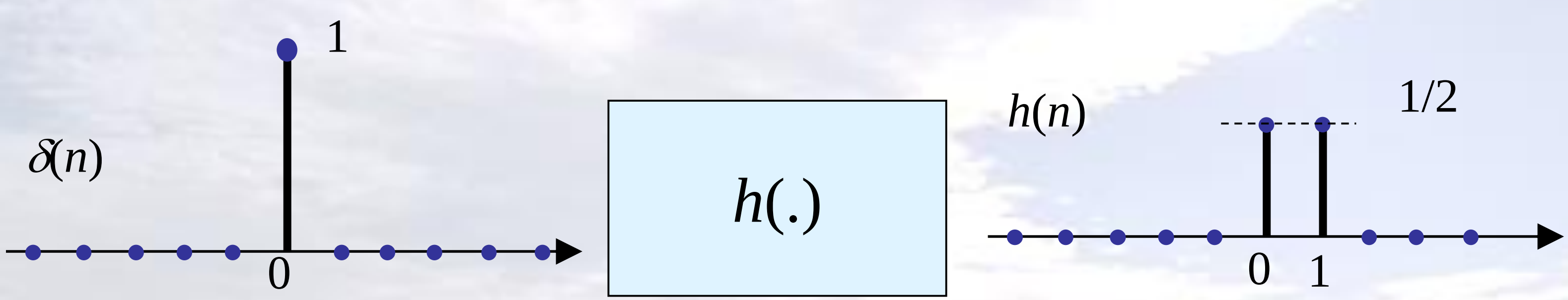
A liner system is defined by
a difference equation:

$$y(n)=\frac{1}{2}x(n)+\frac{1}{2}x(n-1)$$

Impulse response of this system is:

$$h(n)=\frac{1}{2}\delta(n)+\frac{1}{2}\delta(n-1)$$





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Signal	Impulse response
x(0)=1	h(0)=1
x(1)=0	h(1)=1
x(2)=0	h(2)=1

$$y(n)=\sum_{k=-\infty}^{k=\infty} h(k)\,x(n-k)$$

$$|||$$

$$y(n)=\sum_{k=-\infty}^{k=\infty} x(k)\,h(n-k)$$

y(0) = x(0)h(0) + x(1)h(-1) + x(2)h(-2) = 1+0+0 = 1
y(1) = x(0)h(1) + x(1)h(0) + x(2)h(-1) = 1+0+0 = 1
y(2) = x(0)h(2) + x(1)h(1) + x(2)h(0) = 1+0+0 = 1
y(3) = x(0)h(3) + x(1)h(2) + x(2)h(1) = 0+0+0 = 0
y(4) =…………….

Signal Impulse response

Signal	Impulse response
x(0)=1	h(0)=1
x(1)=1	h(1)=1
x(2)=1	h(2)=1

$$y(n)=\sum_{k=-\infty}^{k=\infty} h(k)\,x(n-k)$$

$$\equiv$$

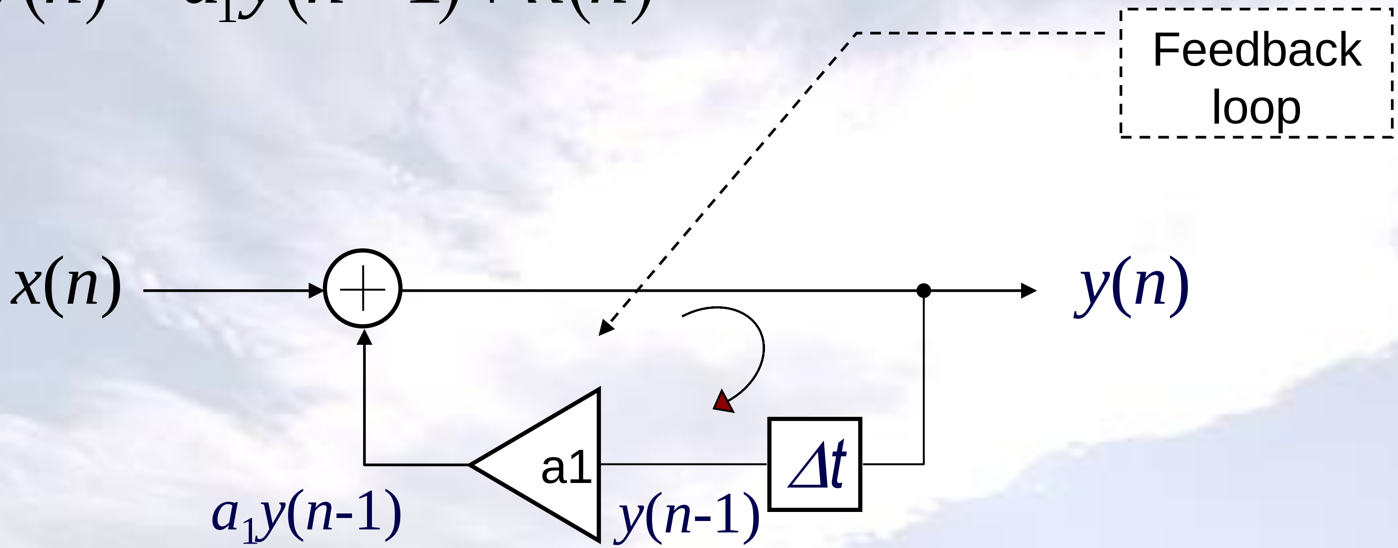
$$y(n)=\sum_{k=-\infty}^{k=\infty} x(k)\,h(n-k)$$

y(0) = x(0)h(0) + x(1)h(-1) + x(2)h(-2) = 1+0+0 = 1
y(1) = x(0)h(1) + x(1)h(0) + x(2)h(-1) = 1+1+0 = 2
y(2) = x(0)h(2) + x(1)h(1) + x(2)h(0) = 1+1+1 = 3
y(3) = ……………..
y(4) =…………….

Response of the linear system to the discrete signal

The system's impulse response can be infinite, eg:

$$y(n) = a_1 y(n-1) + x(n)$$



For what values of a_1 the response of the system is bounded?

Quiz questions

1. Any **discrete time signal** can be defined by?
   a) a linear combination of weighted unit pulse functions
   b) a linear combination of delayed unit pulse functions
   c) a linear combination of unit pulse functions
   d) a linear combination of delayed and weighted pulse functions

2. The **impulse response** is sufficient to completely characterize?
   a) a linear memoryless system
   b) a linear system
   c) a time invariant system
   d) a linear time invariant system

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Convolution as filtering

Example:

Convolve ECG signal with an impulse response:

$$h(n) = \left[\frac{1}{24} \quad \frac{1}{24} \quad \frac{1}{24} \quad \frac{1}{24} \quad \ldots. \quad \frac{1}{24} \right]_{24}$$

$$y(n) = \sum_{k=0}^{k=23} \frac{1}{24} x(n-k)$$

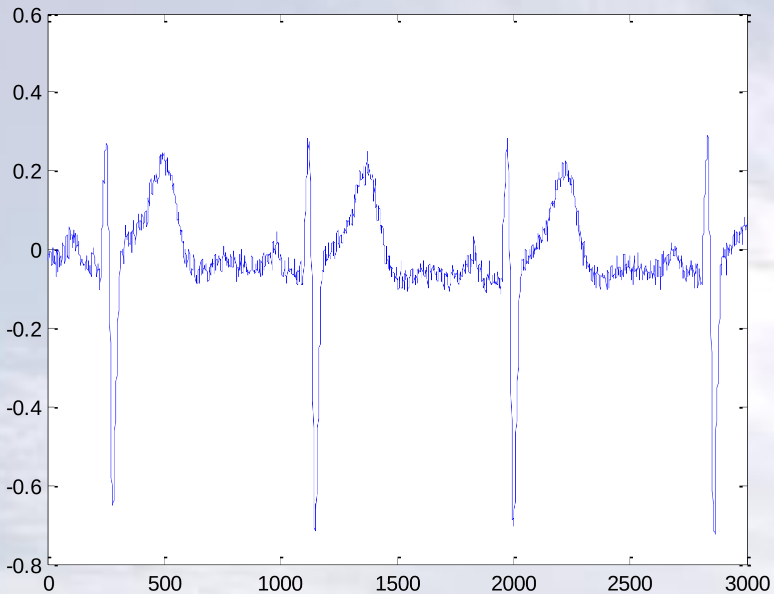
Convolution as filtering

```
# Python (interactive mode)
from scipy.io import loadmat,savemat
ecg=loadmat('ecg_all.mat')['ecg_s']  #load ecg signal and assign to variable ecg
ecg=reshape(ecg,len(ecg))   #reshaping required for .mat files
plot(ecg)
h=1/24.*ones(24)                     #define impulse response h
y=convolve(h,ecg)                    #result of filtering
plot(y)
```

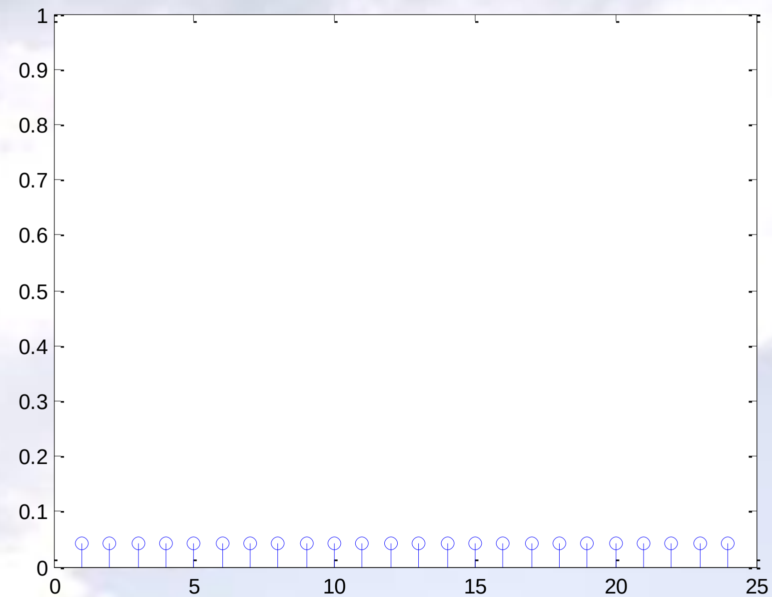
$$y(n)=h(0)x(n)+h(1)x(n-1)+\ldots+h(23)x(n-23)=\sum_{k=0}^{k=23}h(k)x(n-k)$$

Convolution as filtering

ECG signal



Filter’s impulse response



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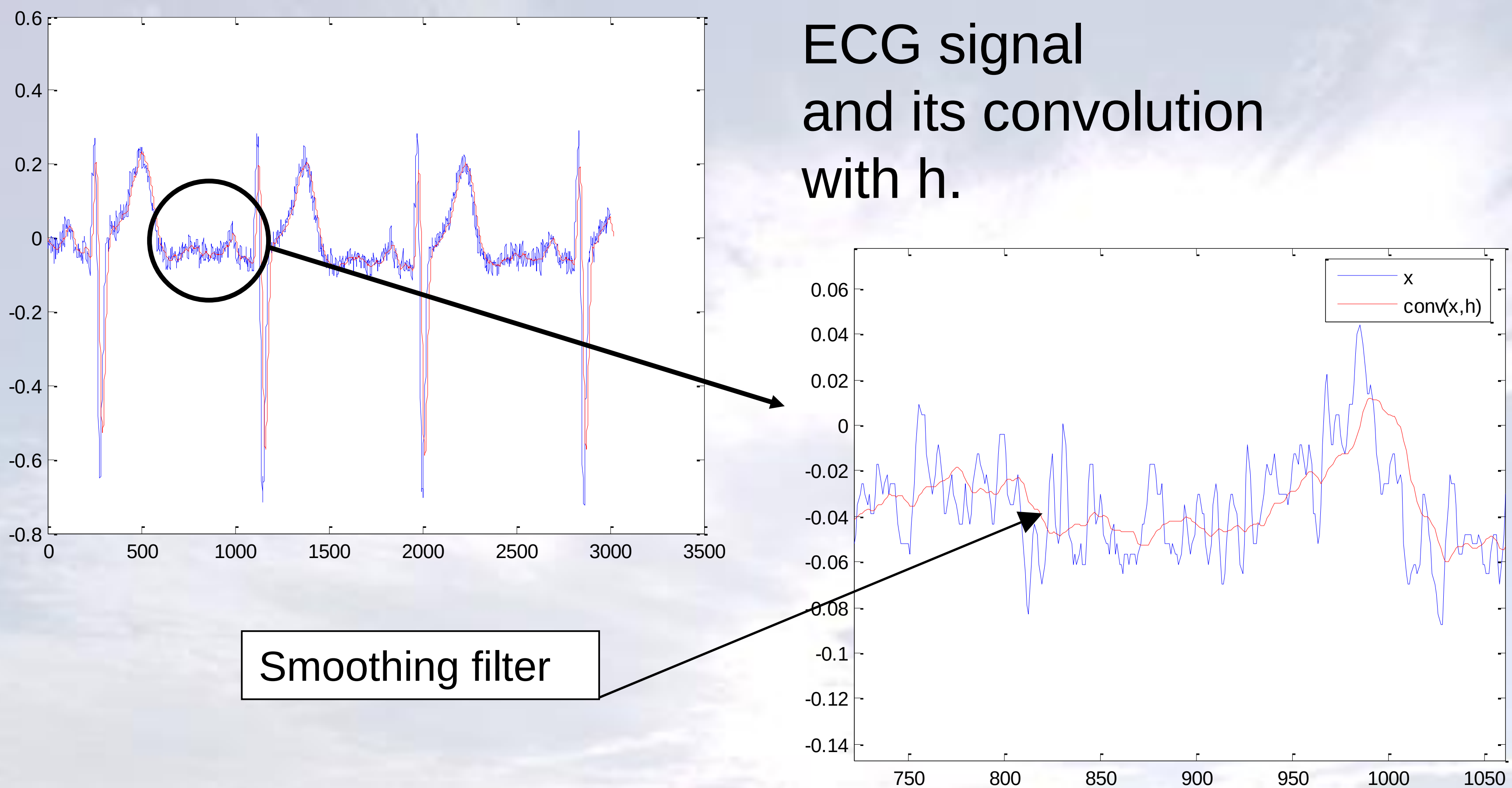
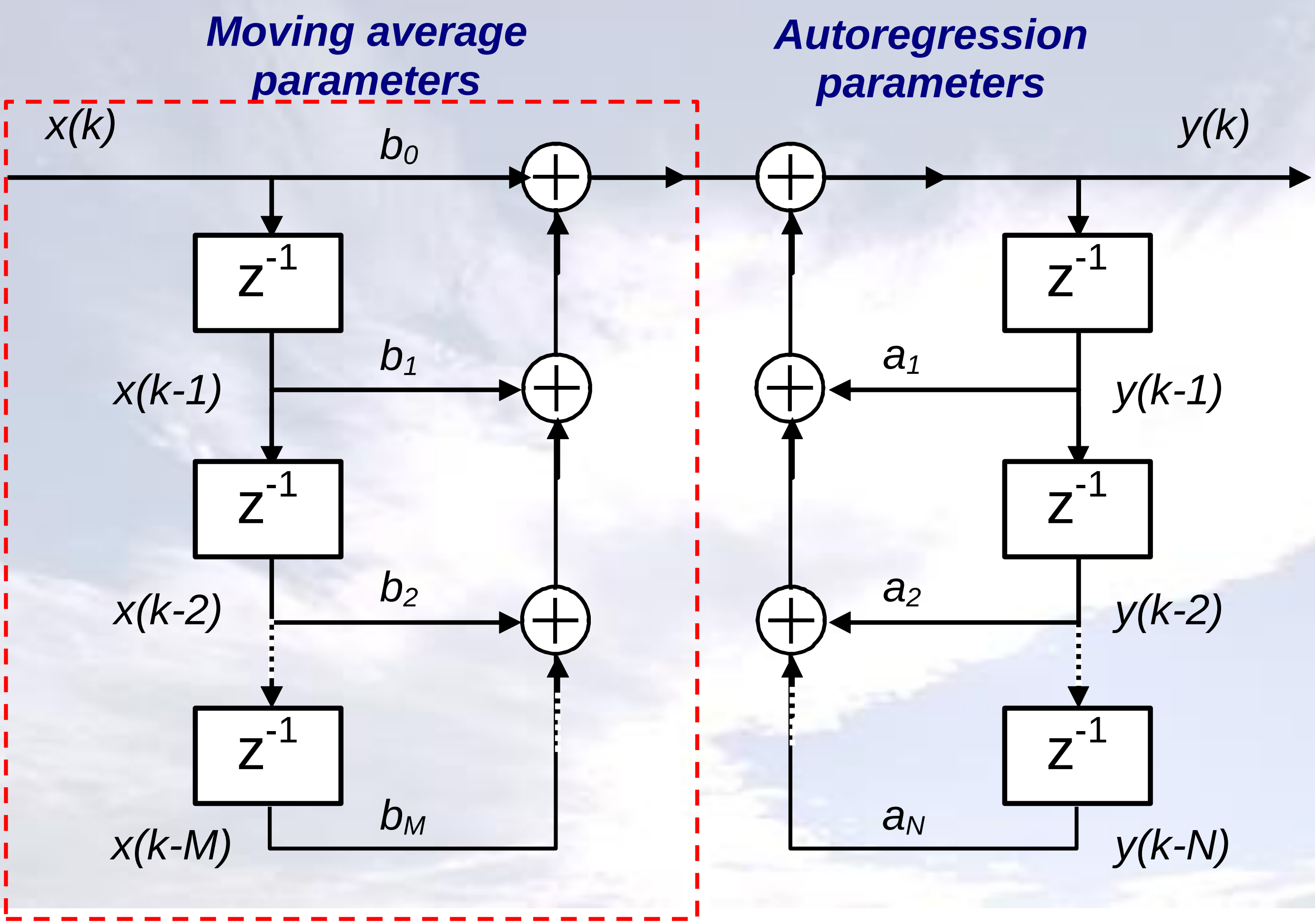


Convolution → digital filters



Systems and convolution - summary

1. Types of systems
2. Various systems’ properties
3. LTI systems
4. Continous vs discrete signals
5. Convolution
   - definition
   - calculation
   - examples - filtering

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„SIGNAL PROCESSING”

Prezentacja multimedialna współfinansowana przez Unię Europejską w ramach Europejskiego Funduszu Społecznego w projekcie pt.

„Innowacyjna dydaktyka bez ograniczeń - zintegrowany rozwój Politechniki Łódzkiej - zarządzanie Uczelnią, nowoczesna oferta edukacyjna i wzmacniania zdolności do zatrudniania osób niepełnosprawnych”

Politechnika Łódzka

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www.kapitalludzki.p.lodz.pl